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In-depth analysis and simulation study of an innovative fuzzy approach for ranking alternatives in multiple attribute decision making problems based on TOPSIS

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ABSTRACT

In this paper, an innovative fuzzy approach for ranking alternatives in multiple attribute decision making problems based on TOPSIS is presented in-depth and studied through simulation comparison with the original method. The TOPSIS method provides the principle of compromise that the chosen alternative should have the shortest distance from the ideal solution and, simultaneously, the farthest distance from the negative ideal solution. However, the TOPSIS method does not always produce results in harmony with this principle due to an oversimplified definition of its aggregation function which does not grasp the contradictory nature of the principle's formulation. Our approach addresses this issue through the introduction of a fuzzy set representation of the closeness to the ideal and to the negative ideal solution for the definition of the aggregation function which is modeled as the membership function of the intersection of two fuzzy sets. This model enables a parameterization of the method according to the risk attitude of the decision maker. Thus, a class of methods is formulated whose different instances correspond to different risk attitudes of the decision makers. In order to define some clear advises for decision makers facilitating a proper parameterization of the method, a comparative analysis of the proposed class of methods with the original TOPSIS method is performed according to well defined simulation techniques. The results of the simulation experiment show on the one hand that there is no direct correspondence between the proposed class of methods and TOPSIS, and on the other hand that it is adequate to distinguish three instances that correspond respectively to risk-averse, risk-neutral and risk-seeking decision makers. Finally, a numerical example pertaining to the problem of service provider selection is presented to illustrate the application of the proposed class of methods and its functioning.

1. Introduction

According to the interpretation of normative decision theory, a decision is defined as the determination of the course of action from a given set of alternatives which maximizes or satisfies certain criteria. Multiple attribute decision making (MADM) refers to problems where the aim is to find the best solution among all feasible alternatives according to the assessment of multiple quantitative and qualitative attributes. Several methods have been proposed to deal with such problems such as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). The TOPSIS method was first developed by Hwang and Yoon [10] and provides the principle of compromise that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the negative ideal solution. The ideal solution is identified with a hypothetical alternative that has the best values for all considered attributes whereas the negative ideal solution is identified with a hypothetical alternative that has the worst attribute values. In practice, TOPSIS has been successfully applied to solve selection/evaluation problems with a finite number of alternatives [11,19,26].

However, the TOPSIS method has two important shortcomings. Firstly, the performance ratings and the weights of the criteria are given as crisp values whereas real decision problems are usually not defined in crisp terms [29]. Human judgments are uncertain in nature and, therefore, it is not appropriate to represent them by precise numerical values. Consequently, many researchers have proposed the use of linguistic assessments and of the theory of fuzzy sets in order to grasp the vagueness that is inherent in the decision making process [4,14,25]. Secondly, the aggregation function of the TOPSIS method does not produce results such that the highest ranked alternative is simultaneously the closest to the ideal solution and the furthest to the negative ideal solution since these conditions can be conflicting [13,14,16]. Although the aggregation function of TOPSIS exhibits some characteristics that are aligned with the principle of compromise of the TOPSIS method it is

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nonetheless defined in a rather oversimplified way as the measure of "the relative distance between a certain alternative and ideal and negative ideal solutions" than on the basis of a stricter and more comprehensive theoretical framework that would be able to grasp the contradictory nature of the principle of compromise of TOPSIS (Sections 2 and 3).

In a previous study [3], we have proposed the use of a method introducing an innovative model for the definition of the aggregation function that is based on a fuzzy set representation of the closeness to the ideal and to the negative ideal solution. This approach models the aggregation function as the membership function of the intersection of two fuzzy sets, i.e. the fuzzy set of alternatives that have "the shortest distance from the ideal solution" and the fuzzy set of alternatives that have "the farthest distance from the negative ideal solution". An initial approach to handle this issue has been proposed in the context of Multiple Objective Decision Making problems (MODM) by Lai et al. [13]. Their model employs the max–min operation for the solution of the corresponding bi-objective problem [2] which is equivalent to the use of the intersection connective of fuzzy sets that was proposed originally by [27]. This intersection connective reflects the largest fuzzy set contained in the two initial sets (since it is defined as the minimum of the membership functions of the original sets). The disadvantage of this formulation is that it does not permit ordering of elements of the intersection of two sets whose minimum values of the membership functions of the original sets are equal, i.e. the decision maker takes into account only the worst characterization of an alternative.

For this reason, we have proposed the use of the class of connectives proposed by Yager [24], which includes Zadeh's connective (and thus Lai's approach) as a special case. Yager's class of connectives permits modeling of the relative importance of membership values as well as modeling of the "strength" of the intersection connective. Thus, it provides the mathematical basis for the modeling of the notion of closeness to the ideal and the negative ideal solution and enables a formal definition of the relation of the closeness to the ideal solution with the closeness to the negative ideal solution. As a result, a class of methods is formulated whose different instances correspond to different risk attitudes of the decision makers.

In order to compare our approach with the original TOPSIS method as well as to define some clear advises for decision makers facilitating the selection of the proper instance of the proposed class of methods, we have designed and conducted a simulation experiment. The results of the simulation experiment show on the one hand that there is no direct correspondence between the original TOPSIS method and our approach, and on the other hand that it is adequate to distinguish three instances of the proposed class of methods that correspond respectively to risk-averse, risk-neutral and risk-seeking decision makers. Thus, the decision maker may be guided along this line of interpretation to select a value of parameter p that better suits his/her risk attitude.

Finally, an illustrative example of a service provider selection problem is examined to demonstrate the application of the proposed class of methods and its functioning, in relation to different attitudes of decision makers towards risk.

The remainder of this paper is organized as follows. Section 2 briefly discusses the original TOPSIS method and its shortcomings as far as the choice of its aggregation function is concerned. The intent of Section 3 is to describe our approach aimed to overcome these shortcomings. In Section 4 we describe the simulation experiment that we conducted in order to study the proposed method. The obtained results are displayed and discussed in Section 5. A numerical example of a service provider selection problem is given in Section 6. Finally, Section 7 concludes and indicates some possible extensions of the results of this research.

2. The TOPSIS method

It is necessary for the clarity of presentation to provide a brief summary of the TOPSIS method which was originally proposed by Hwang and Yoon [10]. TOPSIS ranks alternatives according to their closeness to a hypothetical ideal alternative (zenith) and a hypothetical negative ideal alternative (nadir). In this context, the domain set of alternatives is defined as a n-dimensional Euclidean space. Therefore, each alternative is represented as a point in this space. In order to be able to define the zenith and the nadir points, a basic assumption is that each attribute is characterized by either monotonically increasing or decreasing utility. Then, TOPSIS principle is that solutions are defined as those alternatives which are at the same time farthest from the nadir point and closest to the zenith point, with closeness being measured by the Euclidean distance.

TOPSIS models this principle by defining "relative closeness to ideal solution", C_i^+ (i.e. the relative distance between a certain alternative and ideal and negative ideal solutions) according to the relation:

$$C_i^+ = \frac{S_i^-}{S_i^- + S_i^+} \tag{1}$$

where S_i^- is the n-dimensional Euclidean distance between the ith alternative and the nadir point and S_i^+ is the n-dimensional Euclidean distance between the i$^{\text{th}}$ alternative and the zenith point. According to [10] the procedure of the TOPSIS method consists of five steps:

Step 1 *Construct the normalized decision matrix*:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{2}$$

where x_{ij} is the value of alternative i with respect to attribute j ($1 \le i \le m$ for m alternatives, $1 \le i \le n$ for n attributes).

Step 2 *Construct the weighted normalized decision matrix*:

$$\upsilon_{ij} = w_j r_{ij} \tag{3}$$

Step 3 *Determine the ideal and the negative ideal solutions*:

$$A^+ = \{\upsilon_1^+, \upsilon_2^+, \ldots, \upsilon_n^+\},$$
$$A^- = \{\upsilon_1^-, \upsilon_2^-, \ldots, \upsilon_n^-\}$$

where

$$\upsilon_j^+ = \{\max_i \upsilon_{ij}, j \in J : \min_i \upsilon_{ij}, j \in J'\}, \tag{4}$$

$$\upsilon_j^- = \{\max_i \upsilon_{ij}, j \in J' : \min_i \upsilon_{ij}, j \in J\} \tag{5}$$

where J is the set of "benefit" attributes and J' is the set of "cost" attributes.

Step 4 *Measure the separation of alternatives from the "ideal" solutions*:

$$S_i^+ = \sqrt{\sum (\upsilon_{ij} - \upsilon_j^+)^2}, \tag{6}$$

$$S_i^- = \sqrt{\sum (\upsilon_{ij} - \upsilon_j^-)^2} \tag{7}$$

Step 5 *Calculate the "relative closeness" of each alternative to the ideal solution and the final ranking*: the "relative closeness" of each alternative to the ideal solution is calculated according to Eq. (1).

A more detailed description of the steps of TOPSIS is given in [10].