Dimension reduction in mean-variance portfolio optimization

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A R T I C L E   I N F O

Article history:
Received 4 March 2017
Revised 18 August 2017
Accepted 8 September 2017
Available online 19 September 2017

Keywords:
Non-negative principal components analysis
Non-negative matrix factorization
Multivariate time series
Portfolio backtesting
Statistical variance procedure

A B S T R A C T

Dimension reduction methods are useful pre-processing tools for efficient quantitative analysis with the aim to preserve the main features of the multidimensional data. However, negative values resulting from the transformation may obscure the interpretation of the analysis. This novel study aims to investigate the effects of non-negative dimension reduction methods on the mean-variance portfolio optimization model. Backtesting results for major stock market indices show that reducing dimensionality of asset prices may improve the overall efficiency of the mean-variance portfolio optimization output.

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1. Introduction

A rational investor should perform quantitative analysis to allocate the available funds to various financial assets. The decision to determine the rate of allocation is necessary yet there are countless combinations of assets to invest in. Moreover, the complexity of this decision problem increases as transaction costs, hedging alternatives, and correlations enter the picture. For instance, although financial assets have a tendency to move together, the correlations between their returns are usually imperfect. Considering these mathematical standpoints, modern portfolio theory developed by Markowitz (1952) has framed the optimal portfolio selection problem by presenting the mean-variance (MV) model. This model allocates resources by using diversification during the optimization process.

A portfolio is a weighted combination of assets and the MV model solves for the optimal allocation rates of each asset in a portfolio. The MV model is an optimization process to minimize the portfolio variance at a prescribed rate of return. It quantifies uncertainty by using the standard deviation of assets’ financial returns. Therefore, the covariance matrix of asset returns is a significant tool to determine the risk of a portfolio. Since the risk measure contains the assets’ both independent and interdependent performances, using a covariance matrix achieves a diversified portfolio. The insight of melding asset risk and returns in a portfolio context through the proposal of the modern portfolio theory has established the mean-variance paradigm as an “anchor for portfolio optimization” (Pfaff, 2013).

An optimal portfolio is a portfolio bearing the minimum risk at an expected level of return. Efficient portfolio frontier is the set of all optimal portfolios on where each optimal portfolio lies at a defined risk level. Trade-off between risk and return is a subjective decision depending on an investor's attitude towards risk. Efficient frontier guides the investor to choose an optimal portfolio at a risk level with an expected return.

The growing interest in contemporary dimension reduction methods allow for a fast and thorough analysis while preserving the features of the multidimensional data at adequate and manageable levels. Although considerable research has been devoted to the field, to the authors’ knowledge, a fundamental attempt on investigating the effects of dimension reduction methods applied on the MV portfolio optimization model is not available. This study aims to contribute to the literature by presenting a comparative analysis and application of two fundamental non-negative dimensionality reduction methods, namely the non-negative matrix factorization (NMF) and non-negative principal components analysis (NPCA), on the MV model.

Reducing dimensionality of a multivariate time series of asset prices requires further interpretation as the post analysis stage may inherit negative price values after the transformation. Since asset prices are always positive –similar to data related with image, text, certain economic activities, and other related areas that require non-negativity constraint– reduced dataset with negative values is not a valid input to use in the MV model. Related litera-
ture does not address this problem and requires an agreeable solution. This study presents a perspective to integrate non-negative dimension reduction methods into the MV model. Daily observations of asset prices are reduced to a lower time dimension to capture the market's long-term pattern. Using a non-negative dimension reduction method for reducing the time dimension of the data allows for the representation of non-negative prices at a lower-dimensional space.

To explore the interaction between the reduced asset prices and the MV portfolio optimization model, this paper experiments on different scenarios with short selling and ceiling constraints. The rest of the paper is as follows: Section 2 reviews the MV model and real-world constraints. Section 3 is devoted to two non-negative dimension reduction methods applicable for multivariate time series. Section 4 integrates the non-negative dimension reduction methods prior to the MV model. Section 5 presents the empirical analysis with a discussion, and Section 6 concludes with future research directions.

2. Mean-variance model

The MV portfolio optimization model states that an investor should act according to "probability beliefs"; the past average returns are good estimators of possible future returns and assets' past performances will persist in the future (Markowitz, 1952). Therefore, the variability of past asset returns can be a scale of future uncertainty. Approached by quadratic programming, the MV model treats returns as a random variable. The objective of the quadratic programming formulation of the MV model in Eq. (1) is to find an efficient portfolio that minimizes the risk:

\[
\min_{\mathbf{w}} \mathbf{w}^\top \mathbf{Q} \mathbf{w}
\]

\[
\sum_{i=1}^{n} \mathbf{R}_i w_i \geq E(\mathbf{R})
\]

\[
\sum_{i=1}^{n} w_i = 1
\]

\[
w_i \geq 0, \quad i = 1, 2, \ldots, n
\]

According to the MV model, an optimal portfolio's expected return and variance are as stated in Eqs. (2) and (3):

\[
E(\mathbf{r}_{\text{portfolio}}) = \sum_{i=1}^{n} E(\mathbf{r}_i) w_i
\]

\[
\sigma_{\text{portfolio}}^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{cov}(\mathbf{r}_i, \mathbf{r}_j) w_i w_j
\]

where \(E(\mathbf{r}_i)\) is the expected return of the \(i\)th asset, \(w_i\) is the allocation rate (the weight of the \(i\)th asset in the portfolio), \(\text{cov}(\mathbf{r}_i, \mathbf{r}_j)\) is the covariance between the \(i\)th and \(j\)th assets, and \(n\) stands for the number of assets in the portfolio.

The objective function in Eq. (1) measures the risk by using \(\mathbf{Q}\), the covariance matrix of assets' financial returns. The constraints of the model respectively indicate that the expected return should be greater than a target rate, full investment is required, and short selling is inadmissible. The investor should expect to get different weights for different combinations of mean and variance. All portfolios, which minimize risk for a given expected return, are efficient portfolios that create the efficient frontier.

2.1. Practical constraints

The mean-variance model in Eq. (1) is a basic model which allows only buying for long term. On the other hand, investors face different and complicated scenarios because of practical matters, requirements and regulations (Skolpadungket, Dahal, & Harpornschat, 2007). Typical real-world situations in markets include short selling, floor and ceiling—which are also known as bounding, box, or quantity– constraints, as well as other constraints related to transaction costs, round lots, and cardinality (Chiam, Tan, & Al Mamum, 2008; Crama & Schyns, 2003; Tollo & Roli, 2008). These situations can be implemented in MV portfolio optimization problem as model constraints.

Short selling takes place when an investor sells an asset that she does not own—with the expectation that its price will decrease in the near future. Only then, the investor expects to benefit from buying the asset at a lower price. Complementarily to the traditional type of trading, short selling allows for a bilateral trading opportunity. Kumar, Mitra, and Roman (2008) emphasize the importance of short selling and of forming a long-short model. Jacobs and Levy (2014) state that constructing an integrated long–short model may lead to an increased performance as opposed to a long-only portfolio. Negative allocation rates in the MV portfolio optimization model refer to short positions (Würtz, Chalabi, Chen, & Ellis, 2009). For achieving both long-buying and short–selling within the same portfolio, the long-short portfolio optimization model replaces the last constraint of non-negativity in Eq. (1) with the following constraint:

\[
w_i \in \mathbb{R} \quad \forall \ i, \quad i = 1, 2, \ldots, n
\]

The floor and ceiling constraint is another practice which limits the proportion of assets within a portfolio by imposing lower and upper limits. Assigning lower weights as a limit avoids excessive allocation to assets higher than the predetermined limit and thus reinforces diversification (Tollo & Roli, 2008). The classical long-only model replaces the non-negativity constraint \(w_i \geq 0\) in Eq. (1) with the one that also sets an upper limit \((U)\):

\[
0 \leq w_i \leq U, \quad i = 1, 2, \ldots, n
\]

Since the allocation rates are positive for long and negative for short positions, the ceiling constraint of the long-short model is as follows:

\[
\|w\| \leq U, \quad w_i \in \mathbb{R} \quad \forall i, \quad i = 1, 2, \ldots, n
\]

3. Non-negative dimension reduction

The MV model solves for the optimal asset allocation by using multivariate time series as the input. Time series are basically sequences of data sampled over time (Ruppert & Mateoson, 2015). Pfaff (2013) provides stylized facts of univariate and multivariate time series of financial returns. Univariate time series analysis aims to understand the structure of a single variable whereas analysis of multivariate time series focuses on the interdependency of multiple time series.

Time domain and frequency domain are two complementary modes of time series analysis. Time domain analysis examines data through parametric models using autocorrelation functions. On the other hand, frequency domain analysis describes the fluctuations of data using mathematical transformations (Wei, 2006). In the frequency domain, sequential and multidimensional structure of the time series facilitates pattern discovery. The correlation of data points provides an opportunity for reducing data to a simpler representation without substantial information loss (Zhu, 2004).

Dimension reduction methods either select or extract features to capture a brief overview of the data. Feature extraction methods—e.g. subset selection algorithms—reduce the dimensionality by selecting a subset of features; yet feature extraction methods project the features of the dataset onto a new feature space of lower dimension (Bennasar, Hicks, & Setchi, 2015). PCA, wavelet transform, and random projection are among the major dimension reduction methods for reducing dimensionality in multivariate time series (Keogh, Chakrabarti, & Pazzani, 2001).

Dimension reduction of prices in the MV model directly tackles the complex problem of large-scale portfolio construction.
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