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Quantum information and accounting information: Exploring conceptual applications of topology

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ABSTRACT

Our previous attempt resulted in a paper by the same five authors, "Quantum information and accounting information: their salient features and conceptual applications," published in the July–August 2006 issue of *the Journal of Accounting and Public Policy*. We now extend the previous paper to examine *topological* quantum computation, a remarkably innovative approach to decoherence and imprecise quantum computation. In this approach, exotic topological states are created for a natural medium to store and manipulate quantum information globally throughout the entire system. The process is intrinsically protected against imprecision and decoherence. We also explore conceptual, if not technical, applications of topological quantum computation to accounting. This is done by introducing topology's inherent emphasis of *qualitative* characteristics to traditional accounting which has been dominated by *quantitative* characteristics. Here, financial statements' monetary amounts may be contrasted to internal controls' error frequencies. Part I of the paper deals with applications of topology to quantum information, after a brief introduction to basic tools. In particular the use of Fibonacci anyon and its powerful results are explained. Part II deals with applications of topology to accounting information. Part III deals with applications of topology to other potential fields.

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1. Part I: Applications of topology to quantum information

1.1. Introduction

This paper is a continuing attempt to improve our understanding of accounting as an information source with an eye on new developments in quantum information theory and quantum computation. Here, we focus on *topological* quantum computation and discuss potential applications of topology to accounting system and accounting information. Our earlier attempt (Demski et al., 2006) explored quantum information and identified some conceptual applications of quantum information to accounting.

The Sarbanes-Oxley Act of 2002 has made accounting information more legalistic and blurred the boundary between financial and managerial accounting. This motivates our exploration of topology and its possible implications to accounting as we believe this development will most likely aid an important segment of current accounting information, especially internal controls mandated by the Sarbanes-Oxley Act. Cautions are provided as we recognize it is unclear how the application of topology helps our understanding of accounting information at large, in particular when a manager's opportunistic behavior is a first-order concern. At the same time, we are hopeful the discussion may motivate a search for new ways to tackle difficult issues.

Section 1.2 presents an overview of key concepts in topology. With this background, Section 1.3 elaborates the basics of quantum computation and sketches the possible resolution offered by a topological approach. This completes Part I. We then turn to speculation and our own field of accounting in Section 2.1 to explore accounting applications of topology, ending Part II. We then deal with several features of topology in Section 3.1 that have led to applications in a variety of other fields. Section 3.2 ends Part III with conclusions.

1.2. Some key concepts in topology

To facilitate our understanding of the application of topology in quantum information theory, we discuss a few key concepts in topology.

Topology inherently is a classification system. It deals with properties that are unaltered by smooth deformation, as opposed to by tearing, cutting, or rejoining.[1] We are interested in partitioning the class of spaces into topologically equivalent classes.[2]

Formally, for a non-empty set X, a class T of subsets of X is a topology on X such that (i) $\emptyset$ and X belong to T; (ii) the union of any number of sets in T belongs to T; and (iii) the intersection of any two sets in T belongs to T. The pair (X,T) is called a topological space. Property P of sets is topologically invariant if whenever a topological space (X,T) has P, then every space homeomorphic to (X,T) also has P. Topological spaces (X,T) and (Y,T^*) are said to be homeomorphic if there exists a bijective function f: $X \rightarrow Y$ such that f and f^{-1} are continuous.

Topological spaces, in mathematics, are generalizations of Euclidean spaces in which the idea of closeness, or limits, is described in terms of relationships between sets rather than in terms of distance. Every topological space consists of: (1) a set of points; (2) a class of subsets defined axiomatically as open sets; and (3) the set operations of union and intersection. In addition, the class of open sets in (2) must be defined in such a manner that the intersection of any finite number of open sets is itself open and the union of any, possibly infinite, collection of open sets is likewise open. The concept of limit points is of fundamental importance in topology; a point p is called a limit point of the set S if every open set containing p also contains some point(s) of S (points other than p, should p happen to lie in S) (Bing, 1980, p. 48).[3]

[1] A donut and coffee cup are topologically indistinguishable as one can be identical to the other by means of gradual deformation without cutting or joining.

[2] Two topologically equivalent spaces are also called homeomorphic.

[3] Topology has many approaches. Basener (2006) in Appendix, "Perspectives in Topology," provides five essays on different approaches to topology – point set topology, geometric topology, algebraic topology, combinatorial topology, and differential topology. Basener says "Point set topology is the purest form of topology and provides a basic language for the rest of mathematics, relying on the basic definitions of open sets and topological spaces." Topological quantum computation explores the connection between algebraic topology and braid topology.