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Prediction of Pressure Gradient and Holdup in Small Eötvös Number Liquid-Liquid Segregated Flow*

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Abstract The segregated flow pattern, which occurs in a 26.1 mm diameter, horizontal, stainless steel test section, is investigated. Pressure gradient and *in situ* phase distribution data were obtained for different combinations of phase superficial velocities ranging from 0.05 m·s^{-1} to 0.96 m·s^{-1}. For the current small Eötvös number liquid-liquid system ($Eo_{\mathrm{D}}=4.77$), the dominant effect of interfacial tension and wall-wetting properties of the liquids over the gravity is considered. The approach introduces the closure relationship for the case of turbulent flow in a rough pipe, and attempts to modify the two-fluid model to account for the curved interface. In present flow rates range, wave amplitudes were found small, while interfacial mixing was observed. An adjustable definition for hydraulic diameters of two fluids and interfacial friction factor is adopted. The predicted pressure gradient and in situ phase distribution data have been compared with present experimental data and those reported in the literature.

Keywords two-fluid model, pressure gradient, holdup, liquid-liquid flow, interfacial shear

1 INTRODUCTION

Flows of two immiscible liquids occur over a wide range of volumetric flow rates in pipelines for multiphase flow transport. In particular, in the petroleum industry, mixtures of oil and water are transported in pipes over long distances [1]. In order to design the transport and production systems, prediction of oil-water flow characteristics, such as flow pattern, water holdup and pressure gradient, is required in many engineering applications [2]. Despite their importance, oil-water flows have not been studied to the same extent of gas-liquid flows [3], and only some mature theories for gas-liquid flows are modified for the development of modeling liquid-liquid flows. It is noted that the flow structure of oil-water mixtures in pipes is quite different from that of gas-liquid mixtures. The density difference between oil and water is relatively low [4], while the viscosity ratio encountered extends over a range of many orders of magnitude [5]. Also, the occurrence of shorter interfacial waves and smaller dispersed phase droplets are related to the lower free energy at the interface [6]. Furthermore, oil-water mixtures may show a Newtonian or non-Newtonian rheological behavior as the result of emulsification and dispersion [7, 8]. Therefore, the various concepts and results related to gas-liquid two-phase flows cannot be simply applied to liquid-liquid systems [9].

The main difficulties in modeling oil-water flows arise from the interface between phases and the discontinuity associated with them. Much effort has been given to the flow patterns and their transition, and the corresponding theoretical analysis, similar to Taitel and Dukler's [10] two-fluid model, is commonly used for the prediction of design parameters, such as phase fraction and pressure gradient. Trallero *et al.* [11] observed flow patterns in a 50.8 mm I.D. straight pipe using a refined oil with a viscosity of 28.8 mPa·s and a density of 884 kg·m^{-3}. He identified six oil-water flow patterns and classified them into two categories: segregated flow and dispersed flow. In his study, a complete two-fluid model was used to analyze the stratified/non-stratified transition. Stratified flow was predicted by the viscous Kelvin-Helmholtz analysis while inviscid Kelvin-Helmholtz theory predicted the stratified flow with mixing at the interface. However, the application of two-fluid model is dependent on the interface shape, and much attention should be paid to the closure laws used for the liquid-wall and interfacial shear stresses [12].

In common two-fluid model, the interface separating the two phases under stratified flow is often assumed to be planar. However, extensive and careful observations of the interfacial behavior confirm that the phase interface in segregated flow is usually not flat [13, 14]. For instance, in stagnant two-fluid systems, the interface between phases may approach a plane or lunar configuration depending on the physical properties of the fluids, fluid-wall wettability, the geometrical dimensions and respective fraction of each phase. In oil-water systems, due to the relatively low density differential between the two fluids, the effect of gravity is weakened. Therefore, the wall-wetting properties of liquids and interfacial tension become important and may have a significant effect on the flow pattern. Based on the consideration of a wide range of physical properties encountered in liquid-liquid systems, Brauner *et al.* [15] proposed a non-dimensional Eötvös number to characterize the stratified liquid-liquid flow:

$$Eo_{\mathrm{D}}=\frac{(\rho_2-\rho_1)gD^2}{8\sigma_{12}} \tag{1}$$

For systems with $Eo_{\mathrm{D}}<1$ which resemble the micro-gravity two-phase flow, interfacial tension plays a definite role, whereas liquids wettability with pipe material has considerable effect on the contact angle

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between the liquids and pipe wall. On the other hand, the systems with $Eo_D \gg 1$ correspond to the liquids with a finite density difference and sufficiently large pipe diameter. In such systems, a plane interface can be assumed, and the analysis is similar to common approach for gas-liquid flow. For instance, exact solutions for laminar stratified flows assuming a plane interface between the fluids were obtained by Biberg and Halvorsen [16]. These solutions are valid only for large Eo_D systems. It should be pointed that Brauner *et al*. [15] did not provide specific Eötvös number applicable for planar interface model. To our knowledge, the method with planar interface assumption is applicable to the case of $Eo_D \geqslant 10$ for oil-water flow, while for the systems with $Eo_D < 10$, exactly, $Eo_D \leqslant 5$, this assumption causes significant deviation from actual interface geometry. Besides, liquid-wall and interfacial friction factor, as the closure relations of two-fluid model, are affected by estimated wetted-wall and interfacial perimeter. Consequently, noticeable disagreement exists between the predicted and measured water holdup and friction pressure drop. Therefore, the curved interface model is introduced to current oil-water flow analysis ($Eo_D = 4.77$).

Angeli and Hewitt [17] identified the flow patterns for co-current oil-water flow in horizontal acrylic and stainless steel pipes using high speed video recording and high frequency impedance probes. Their conclusion is that the pipes of the different materials give rise to considerable difference in phase distribution and pressure gradient. On the one hand, the pipe surface roughness affects flow behavior in the fully rough (turbulent) flow regime where flow rates are typically high enough that the inertial forces are dominant and the importance of the viscous sub-layer is diminished, while it does not in the partially rough (turbulent) flow regime. On the other hand, liquids wettability with pipe material has effect on the contact angle between the liquids and pipe wall, and consequential oil-water interface geometry. Therefore, in the present work, both experimental and theoretical studies on oil-water segregated flow in horizontal pipe are carried out. The approach developed for this purpose comprises a two-fluid model with curved interface. Both pipe roughness and liquids wettability with pipe material have been considered.

2 EXPERIMENTAL FACILITY

Data during the horizontal oil-water flow were obtained in a multiphase flow experimental loop. The working fluids were tap water and diesel oil, with the properties shown in Table 1. Note that the wall wettability angle (α) by water is 40° for current stainless steel pipe. Oil was pumped from the oil-water separator by a screw pump to the mixer through a 30 mm I.D. steel pipe, and water was driven by a centrifugal pump. The test section consisted of a 9.7 m long, 26.1 mm internal diameter stainless steel pipe, and a 1.5 m long acrylic pipe for visual observation. The two-phase flow developed over a length of 10.5 m (402 D) and arrived at the entrance of the test section. Two locations for pressure gradients were configured along the test section as depicted in Fig. 1. A variable-reluctance differential pressure transducer with a full scale accuracy of ± 0.2 % was adopted. The range of the transducers was consistent with the expected pressure drop (from theoretical prediction).

Table 1 Physical properties of water and diesel oil
(20ºC, 0.1 MPa)

Fluid	Density /$kg\cdot m^{-3}$	Viscosity /mPa·s	Interfacial tension/$N\cdot m^{-1}$	Eötvös number
tap water	998	0.97	0.028	4.77
diesel oil	838	3.47		

The *in situ* phase distribution was characterized by the height of water climbing along the wall circumferentially and the height of water layer at the vertical plane passing the pipe axis, which were measured by two sets of different conductance probes. Each set included parallel chromel wires and parallel ring probes with the spacing of 40 mm. A probe, consisting of two chromel wires, traversed the diameter of the pipe vertically (Fig. 2). The parallel wires behaved like a pair of parallel cylinders separated by a fixed distance of 1.3 mm. One of the wires was excited with a high frequency alternating voltage inducing a current through the probe that was dependent on the height of water layer between the wires [18]. The parallel ring probes [19], as shown in Fig. 3, were composed of a pair of brass rings with the thickness of 4 mm, and these rings were embedded flush with the inner surface of pipe covered by insullac. Nonconductive acrylic resin with the axial thickness of 10 mm was filled between the parallel rings. Both probes were statically calibrated by locating the depth of probes submerged by water.

Images for clarification of the flow pattern were

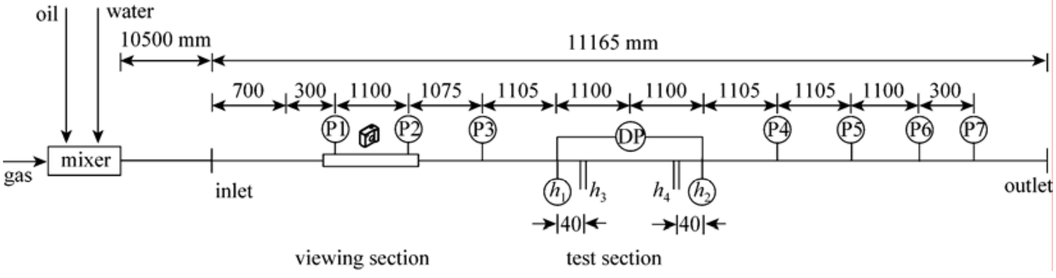

Figure 1 Diagram of test section and viewing section

‖ parallel wire conductance probe; ⓗ parallel ring probes; DP pressure differential transducer; Ⓟ pressure transducer