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THE PRESSURE TRANSIENT ANALYSIS OF DEFORMATION OF FRACTAL MEDIUM*

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Abstract: The assumption of constant rock properties in pressure-transient analysis of stress-sensitive reservoirs can cause significant errors in the estimation of temporal and spatial variation of pressure. In this article, the pressure transient response of the fractal medium in stress-sensitive reservoirs was studied by using the self-similarity solution method and the regular perturbation method. The dependence of permeability on pore pressure makes the flow equation strongly nonlinear. The nonlinearities associated with the governing equation become weaker by using the logarithm transformation. The perturbation solutions for a constant pressure production and a constant rate production of a linear-source well were obtained by using the self-similarity solution method and the regular perturbation method in an infinitely large system, and inquire into the changing rule of pressure when the fractal and deformation parameters change. The plots of typical pressure curves were given in a few cases, and the results can be applied to well test analysis.

Key words: stress-sensitive reservoir, fractal, permeability modulus, pressure analysis

1. Introduction

Numerous experiments have proved that the formation of oil reservoir and the fracture network distribution of fractured reservoir are fractal structures. Therefore, the flow theory of fractal reservoir has been developed and applied to oilfield. Fluid flow in hydrocarbon reservoirs and ground water aquifers have been traditionally studied by assuming the formation permeability is constant[1-6]. These assumptions in the fluid flow analysis have given good results in many situations, but with increasing exploitation of petroleum and geothermal resources from low-permeability and fractured formations, these assumptions need to be re-evaluated. Kikani[7] presented the flow model for cylindrical flow systems of deformed media. A perturbation technique was applied to determine the approximate solution and analyze the flow characteristics of deformed media reservoir. Yeung[8] considered the spherical flow problem of deformed media reservoir. A simple technique was applied to obtain approximate solutions, but the error is large. The generalized pseudo pressure function was introduced to characterize the gas flow in pressure-sensitive reservoir[9]. The flow analysis for stress-sensitive reservoirs with double porosity was studied[10-16]. But the flow analysis for stress-sensitive fractal reservoirs has not been performed. Radial and spherical flow in homogeneous reservoir are special cases of fractal reservoir[1] ($d_f = 2$ or 3 , and $\theta = 0$). In this article, the fractal and deformed characteristics of stress-sensitive reservoir are considered. A permeability modulus is introduced to

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Boigraphy: ZHANG Yi-gen (1965-), Male, Ph. D. Student, Senior Engineering derive the radial flow equation for the stress-sensitive fractal reservoir. The perturbation solutions for a constant pressure production and a constant rate production of linear-source well are obtained by using the self-similarity solution method and the regular perturbation method in an infinitely large system, and inquire into the changing rule of pressure when the fractal and deformation parameters change.

2. Flow equation

The following assumptions are made in constructing the mathematical model:

(1)The permeability is stress-sensitive , that is, it depends on pore pressure.

(2)The porous medium is the fractal system with similar structure, the fractal permeable network embedded in impermeable Euclidean matrix, where the fractal network dimension is d_f , and the Euclidean matrix dimension is ($d = 1,2,3$).

The permeability modulus is defined as[1]

$$\gamma = \frac{1}{k}\frac{\mathrm{d}k}{\mathrm{d}p} \tag{1}$$

The parameter γ plays a very important role in the system where changes in effective stress affect the permeability. Basically, it measures the dependence of formation permeability on pore pressure. For practical purpose, γ can be assumed as a constant.

Thus the permeability of fractal reservoir varies exponentially with pore pressure

$$k = k_0 \mathrm{e}^{-\gamma(p_0 - p)} \left(\frac{r}{r_w} \right)^{d_f - \theta - d} \tag{2}$$

where k_0 , p_0 are initial permeability and initial pressure respectively, r, r_w are the radial distance from well and the radius of wellbore respectively, and θ is the fractal diffusion exponent.

The continuity equation for the flow of a single–phase liquid in an isotropic and fractal reservoir can be given by assuming fluid to be slightly compressible and using Darcy's law, which is

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho \frac{k}{\mu}\frac{\partial p}{\partial r}) = \frac{\partial(\phi\rho)}{\partial t} \tag{3}$$

where $\rho = \rho(p)$ is density, and μ is viscosity.

Expanding Eq.(3) under the assumption of pressure-dependent rock properties leads to

$$\frac{\partial^2 p}{\partial r^2} + \frac{\beta}{r}\frac{\partial p}{\partial r} + (\gamma + c_L)\left(\frac{\partial p}{\partial r}\right)^2 =$$

$$\frac{\phi_0 \mu}{k_0}\left(\frac{r}{r_w}\right)^{\theta} (c_L + c_{ma}) \mathrm{e}^{-(c_{ma} - \gamma)(p_0 - p)} \frac{\partial p}{\partial t} \tag{4}$$

where

$$c_L = \frac{1}{\rho}\frac{\partial \rho}{\partial p}, \quad c_{ma} = \frac{1}{\phi}\frac{\partial \phi}{\partial p},$$

$$\phi = \phi_0 \mathrm{e}^{-c_{ma}(p_0 - p)} \left(\frac{r}{r_w}\right)^{d_f - d}$$

Then, assume $\gamma >> c_L$, then Eq.(4) becomes

$$\frac{\partial^2 p}{\partial r^2} + \frac{\beta}{r}\frac{\partial p}{\partial r} + \gamma\left(\frac{\partial p}{\partial r}\right)^2 =$$

$$\frac{\phi_0 c_t \mu}{k_0}\left(\frac{r}{r_w}\right)^{\theta} \mathrm{e}^{\gamma(p_0 - p)} \frac{\partial p}{\partial t} \tag{5}$$

where

$$c_t = c_L + c_{ma}$$

For the case of production of fluid at a constant rate from an infinite reservoir into wellbore, the dimensionless groups are defined as

$$p_D = \frac{2\pi k_0 h(p_0 - p)}{\mu q}, \quad t_D = \frac{k_0 t}{\phi_0 \mu c_t r_w{}^2},$$

$$r_D = \frac{r}{r_w}, \quad \alpha_D = \frac{\mu q \gamma}{2\pi k_0 h}$$

Under the constant-pressure production condition, the dimensionless groups are defined as

$$p_D = \frac{p_0 - p}{p_0 - p_w}, \quad t_D = \frac{k_0 t}{\phi_0 \mu c_t r_w{}^2},$$

$$r_D = \frac{r}{r_w}, \quad \alpha_D = \gamma(p_0 - p_w)$$