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Interfaces with Other Disciplines

Multivariate FX models with jumps: Triangles, Quantos and implied correlation

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ABSTRACT

We propose an integrated model of the joint dynamics of FX rates and asset prices for the pricing of FX derivatives, including Quanto products; the model is based on a multivariate construction for Lévy processes which proves to be analytically tractable. The approach allows for simultaneous calibration to market volatility surfaces of currency triangles, and also gives access to market consistent information on dependence between the relevant variables. A successful joint calibration to real market data is presented for the particular case of the Variance Gamma process.

1. Introduction

The aim of this paper is to introduce an extended multivariate model for FX rates and equity indices based on Lévy processes, with the aim of recovering market consistent information on the correlation between financial assets using suitable derivatives contracts.

The interest in market implied metrics of correlation is motivated by the fact that correlation risk is attracting interest for hedging and regulatory purposes. This risk is in fact present in the trading books of a wide range of buy and sell side market participants, such as bank structuring desks and hedge funds for example. Further, the Basel III supervisory regime (Basel, 2010) is focusing in particular on the impact of wrong-way risk effects on the quantification of counterparty credit risk through metrics such as Credit Value Adjustment (CVA), wrong-way risk denoting the dependence between the counterparty credit worthiness and the value of the investor's position. Capturing correlation risk requires both suitable models for the joint distribution of the relevant variables, and easy-to-implement procedures for the quantification of the parameters controlling the behavior of the joint distribution of choice. Specifically, regarding the latter issue, we note that possible information sources are either past observed values of the variables in question, or derivatives whose quoted price offers an estimate of the market perception of correlation. The estimation of historical correlation from time series though is significantly affected by the length of the sample, the frequency of observation and the weights assigned to past observations. Further, as historical measures are backward-looking, they do not necessarily reflect market expectations of future joint movements in the financial quantities of interest, which are instead necessary for the assessment of derivatives positions and related capital requirements. Alternatively, over the past few years the CBOE has made available daily quotes of the CBOE S&P 500 Implied Correlation Index (Chicago Board Options Exchange, 2009), which replaces all pairwise correlations with an average one. Although this index in general reflects market capitalization, it might not be suitable for example for pricing and assessing counterparty credit risk, due to the equi-correlation assumption.

In light of the previous considerations, our analysis is based on traded multivariate derivative products linked to the existing level of correlation. In particular, we focus on the case of the FX market; due to the presence of currency triangles, liquid options on FX rates, including cross rates, and more sophisticated structures such as Quanto products, the FX market does indeed offer a wide range of derivatives contracts which are exposed to correlation risk and, at the same time, supported by sufficient liquidity. Quanto products are, in fact, financial products with a payoff paid in a different currency from the one in which the underlying asset is traded, allowing investors to participate in the assets profit without facing any exposure to foreign exchange rate risk. Due to these features, such contracts are particularly popular in those markets

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in which the provision of investments in foreign assets is tightly governed by exchange control regulations; this is for example the case in South Africa where commodity investments, such as crude oil, must be listed and settled in South African Rand, although the commodity itself is a dollar-denominated asset.

The choice of using Lévy processes as building blocks for the multivariate FX model is justified by the following considerations. In first place, as reported in the literature, implied correlation – similarly to implied volatility – shows skew patterns (see Da Fonseca, Grasselli, and Tebaldi, 2007; Ballotta and Bonfiglioli, 2016, and references therein, for example) which are not fully consistent with the standard framework based on the Brownian motion, i.e. the Gaussian distribution. Lévy processes represent a simple but effective way of replacing the Gaussian distribution, as many analytical formulas established for models based on the Brownian motion can be easily extended to this more general class of processes. Secondly, we note that the features of asymmetry and excess kurtosis typical of the distributions generated by Lévy processes are consistent with empirical evidence provided for example by Carr and Wu (2007). Thirdly, for a consistent pricing of FX derivatives the multivariate model of choice needs to show symmetries with respect to inversion and triangulation (see De Col, Gnoatto, and Grasselli, 2013, for example); as Lévy processes are invariant under linear transformation, the required symmetries are therefore automatically preserved. Application of Lévy processes for FX modeling at univariate level is relatively well established in the literature, see for example Eberlein and Koval (2006) and references therein.

Multivariate constructions for Lévy processes have attracted interest in the literature over the past few years, for example for modeling and pricing counterparty credit risk (see Lipton and Sepp, 2009; Ballotta and Fusai, 2015, for example). Although several approaches are available, for a detailed survey of which we refer for example to Itkin and Lipton (2015), Luciano, Marena, and Semeraro (2016) and references therein, in the following we adopt the factor construction of Ballotta and Bonfiglioli (2016), in which the overall risk is split in two components: a systematic one originated by sudden changes affecting the whole market (which is also consistent with the results of Atanasov & Nitschka, 2015), and an idiosyncratic one capturing instead shocks originated by asset specific issues. This factor construction also implies that the model shows a flexible correlation structure, a linear dimensional parameter complexity, and readily available characteristic functions, which guarantee a high ease of implementation, and facilitate an integrated calibration procedure providing access to information on the dependence structure between the relevant components. We point out that although our framework is based on the model of Ballotta and Bonfiglioli (2016), in which convolution conditions required to recover a known distribution for the margin processes are derived and applied, our model does not need these restrictive conditions, as they are not necessary to retain its mathematical tractability and a limited number of parameters. As observed for example by Eberlein, Frey, and von Hammerstein (2008), in fact, the presence of convolution conditions aimed at separating the behavior of the margin processes from the correlation structure, although intuitive, leads to a biased view of the dependence in place and reduces the flexibility of the factor model as it fails to recognize the different tail behavior shown by the components of any given multivariate vector. This particular feature distinguishes this approach from the constructions based on multivariate subordinators as for example in Luciano et al. (2016).

In light of the discussion above, this paper offers the following contributions. Firstly, we develop a Lévy processes-based multivariate extended FX framework, which includes additional names to cater for the underlying assets of Quanto products such as Quanto futures and Quanto options. The proposed framework is very general as it can be applied to any class of Lévy processes admitting closed form expressions for their characteristic function. Secondly, we show that the part of the framework concerning the multivariate FX model satisfies symmetries with respect to inversion and triangulation. We note that although these properties are important in order to guarantee a fully consistent FX model, it is not trivial to preserve them once we move out of the standard Black–Scholes framework to allow for more realistic stylized features; for further details on this matter, we refer for example to De Col et al. (2013). Concerning non-Gaussian frameworks for Quanto products, we cite amongst others Branger and Muck (2012), who offer an integrated pricing approach for both Quanto and plain-vanilla options on the stock as well as the foreign exchange rate based on Wishart processes. Thirdly, the proposed model leads to analytical results (up to a Fourier inversion) for the price of both vanilla and Quanto options, which allow for efficient calibration to market quotes in almost real time for both FX triangles and Quanto products. Finally, our model gives access to analytical formulae for the correlation coefficient and the indices of tail dependence, which facilitate the recovery of market implied correlation and the assessment of joint movements on the risk position of investors.

In Section 2, we review the general features of the factor-based multivariate Lévy processes, with particular attention to the results required for the construction of the multivariate FX model, which is introduced in Section 3. In Section 3, we also introduce calibration procedures based on FX triangles and Quanto futures. The numerical analysis is offered in Section 4 together with some considerations on implications on risk management and capital requirements. Section 5 concludes. All the proofs are deferred to the on-line companion.

2. Preliminaries: multivariate Lévy processes via linear transformation

The aim of this section is to provide a comprehensive review of the main results regarding multivariate Lévy processes obtained by linear transformation, which will be used for the construction of the multivariate FX model of Section 3.1, and the pricing results offered in Sections 3.2 and 3.3.

Consider a filtered probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\right)$. Let $\mathbf{L}(t)$ be a Lévy process in $\mathbb{R}^n$, then in virtue of the celebrated Lévy–Khintchine representation its characteristic function is $\phi_{\mathbf{L}}(\mathbf{u}; t) = e^{t\varphi(\mathbf{u})}$ with

$$\varphi(\mathbf{u}) = i\langle \gamma, \mathbf{u}\rangle - \frac{1}{2}\langle \mathbf{u}, \Sigma\mathbf{u}\rangle + \int_{\mathbb{R}^d} (e^{i\langle \mathbf{u},\mathbf{x}\rangle} - 1 - i\langle \mathbf{u}, \mathbf{x}\rangle 1_E(\mathbf{x}))\kappa(d\mathbf{x}), \tag{1}$$

where $\gamma \in \mathbb{R}^n$, Σ is a symmetric, non-negative definite $n \times n$ matrix capturing the variance/covariance matrix of the Gaussian component, $E = \{\mathbf{x} : |\mathbf{x}| \leq 1\}$, and κ is a positive measure on $\mathbb{R}^n$ such that

$$\kappa(\{0\}) = 0, \quad \int_{\mathbb{R}^d} \left(|\mathbf{x}|^2 \wedge 1\right)\kappa(d\mathbf{x}) < \infty.$$

The triplet (γ, Σ, κ) represents the generating triplet of $\mathbf{L}(t)$ and $\varphi(\cdot)$ denotes the characteristic exponent. For the purpose of the financial model put forward in the following sections, we require in particular the finiteness of the moments of the processes of interest; this is guaranteed if each component of $\mathbf{L}(t)$ satisfies

$$\int_{|x|>1} |x|^p \kappa(dx) < \infty \quad p \in \mathbb{R}^+ \tag{2}$$