



# Design of optimal strictly positive real controllers using numerical optimization for the control of flexible robotic systems

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## Abstract

The design of optimal strictly positive real (SPR) controllers using numerical optimization is considered. We focus on how to parameterize the SPR controllers being optimized and the effect of parameterization. Minimization of the closed-loop  $\mathcal{H}_2$ -norm is the optimization objective function. Various single-input single-output and multi-input multi-output controller parameterizations using transfer functions/matrices and state-space equations are considered. Depending on the controller form, constraints are enforced (i) using simple inequalities guaranteeing SPRness, (ii) in the frequency domain or, (iii) by implementing the Kalman–Yakubovich–Popov Lemma. None of the parameterizations we consider foster an observer-based controller structure. Simulated control of a single-link and a two-link flexible manipulators demonstrates the effectiveness of our proposed controller optimization formulations.

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## 1. Introduction

The passivity theorem is one of the most celebrated results in input–output systems theory. In general, a passive system is one that does not generate energy and a very strictly passive system is one that dissipates energy. The passivity theorem states that a passive system and a very strictly passive system connected in a negative-feedback loop are input–output stable [1]. This is

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an extremely powerful statement in the context of nonlinear control; stability of a nonlinear yet passive plant is guaranteed via control in the form of a very strictly passive operator.

In the context of passive mechanical systems, inputs are forces and outputs are rates such as velocity. Flexible structures possess a large number of vibration modes, and in robotics applications have dynamics that are nonlinear. Flexible robotic manipulators are known to be passive via collocation of the joint torques and the angular velocity sensors, that is when the input–output map is between the joint torques and joint angular velocities. The passivity property for these collocated systems is independent of the system mass, stiffness, and modeled vibration modes. The noncollocated map between joint torques and end-tip velocity of a manipulator is not passive, but the map between a modified set of joint torques and a modified output, known as the  $\mu$ -tip rate, has been shown to be passive, thus facilitating passivity-based control of the  $\mu$ -tip rate [2].

Passive and very strictly passive systems that are linear and time-invariant (LTI) are closely related to positive real (PR) and strictly positive real (SPR) transfer functions or matrices [3]. The robust stability of nonlinear flexible robotic manipulators is assured via the passivity theorem when the controllers employed are SPR. In particular, spillover instabilities are avoided. In light of this important stability result, many authors have attempted to formulate rate controllers such that they are SPR. Benhabib et al. [4] suggested the use of SPR rate controllers to control large space structures where the controllers considered were not observer-based. Similarly, McLaren and Slater [5] investigated implementing positive real LQG controllers for the control of large space structures. Lozano-Leal and Joshi [6] investigated the design of LQG controllers, constraining the LQG weight matrices such that the resultant optimal controllers remain SPR. Haddad et al. [7] extend the work of Lozano-Leal and Joshi [6] to include an  $\mathcal{H}_\infty$  performance bound on the closed-loop, again by constraining the appropriate weighting matrices.

The use of numerical optimization algorithms to find optimal SPR controllers has been considered in various papers. In Germeol and Gapsik [8] the design of observer-based SPR compensators using convex numerical optimization was considered. Using linear matrix inequality (LMI) constraints, the  $\mathcal{H}_2$ -optimal control problem was retooled to yield SPR controllers. The controllers were full order, meaning that the controllers and the plant model to be controlled have the same number of system states. Shimomura and Pullen [9] extended the work of Germeol and Gapsik [8], considering the use of iterative algorithms that overcome bilinear matrix inequality issues within the optimal SPR optimization formulation. Again, the resultant controllers were observer-based and full order.

In both Germeol and Gapsik [8] and Shimomura and Pullen [9] the observer gains were those found via the solution to the unconstrained  $\mathcal{H}_2$ -optimal control problem. In Damaren [10], the optimization of single-input single-output SPR controllers of varying order was considered. The SPR controllers were not full order, nor observer-based compensators. Simple inequality constraints in the frequency domain via a transformation from the  $s$ -domain to the  $z$ -domain guaranteed SPRness. In Damaren et al. [11], optimal SPR controllers that approximate a given observer-based, full order controller were found by solving a quadratic programming problem with linear inequality constraints. In Henrion [12] a method using LMIs is presented whereby a transfer function can be designed to be robustly rendered SPR given a Hurwitz denominator polynomial.

Other than the work of Benhabib et al. [4] and Damaren [10], the existing SPR design schemes (that is, optimal design schemes) yield controllers that are observer-based, and thus have the same order as the plant. It remains an open question as to whether or not the optimal SPR controller that solves the  $\mathcal{H}_2$  control problem is observer-based, or even should be the same

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