Imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand

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ABSTRACT

In this article, an economic production quantity (EPQ) model with imperfect production system and advertisement dependent demand has been presented. The advertisement rate has been assumed to be a function of time which has been increased with respect to time at a decreasing rate i.e., it has grown exponentially with respect to time but rate of growth gradually has decreased. Here, the rate of producing defective units has been followed to be a function of production rate. Also, the produced units have been inspected in order to screen the defective units but the screening rate is less than or equal to the production rate and greater than the demand rate. For the developed EPQ model, the total profit has been maximized to obtain the optimum production rate and production run time in the system. Here, algorithms have been developed for finding the optimal profit of the imperfect production inventory model. Finally, different numerical examples have been considered to illustrate the feasibility of the model taking different special cases in the system and then some sensitivity analyses have been carried out to get the impact of some parameters on the objective function of the model.

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1. Introduction

The origin of the Economic Production Quantity (EPQ) model can be traced back to 1918, when E.W. Taft made an extension to the Economic Order Quantity (EOQ) model developed by Harris (1915). The EPQ model is commonly used in the manufacturing sector of the business world to determine the optimal production quantity which optimizes the objective function in the system.

There exists a considerable amount of research works such as Cho (1996) and Goyal and Gunasekaran (1995) in which the items to be produced in a manufacturing system had been considered as perfect. But, this is not a realistic assumption because, in any manufacturing system, the production of defective units is a natural phenomenon to occur from the different difficulties arisen in a long-run production process. Now, the defective items can be treated as a result of imperfect production. At first, Rosenblatt and Lee (1986) considered such type of items in a imperfect production system. After that, Salameh and Jaber (2000) presented a modified

EPQ model that accounts imperfect quality items. Then Hu, Zheng, Guo, and Ji (2010), Khan, Jaber, Guiffrida, and Zolfaghari (2011), Krishnamoorthi and Panayappan (2013), Tripathy and Pattnaik (2013), Sivashankari and Panayappan (2014), Karimi-Nasab and Sabri-Laghaie (2014), Karimi-Nasab and Fatemi Ghomi (2012) and others also worked on EPQ models considering the imperfect production.

Now from the literature survey on imperfect production inventory models, it is seen that there exist two classes of the models on the basis of inspection methods to sort out the defective units from the perfect one. In one class of research, it has been studied that over time the produced items deteriorate in manufacturing system. In this field, the different researchers (Jaber, Bonney, & Moualek, 2009; Kim, Hong, & Chang, 2001; Lee & Park, 1991; Lee & Rosenblatt, 1987; Lin, Chen, & Kroll, 2003; Manna, Das, Dey, & Mondal, 2016; Rahim, 1994; Rahim & Ben-Daya, 2001; Tai, 2013) examined an inspection method on the produced items based on deteriorating production process. Then, Guu and Zhang (2003) used an entire lot of inspection at the end of a process. After that, an inspection of the last *k* units had been used by Yeh and Chen (2006). On the other hand, in another class of research the imperfect production process has been investigated in which the defective units are produced during the production time due to

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machinery fault, labor, raw materials, etc. In this field, there are also many research papers in which different inspection methods had been used by Hsu and Hsu (2013), Cheng (1991), Salameh and Jaber (2000), Chiu (2003), Tripathy and Pattnaik (2013), Hayek and Salameh (2001), Chiu, Lin, and Cheng (2006). Zhang and Gerchak (1990) used an inspection of a fraction of a lot in EOQ model. Recently, Manna, Dey, and Mondal (2014) have developed a 'Three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment' with continuous screening process.

In the classical inventory model, Harris (1915) considered an EOQ model with constant demand rate. Later, this model was discussed by Wilson (1934). Silver and Meal (1969) extended the EOQ model for varying demand rate. Then Donaldson (1977) and Lo, Tsai, and Li (2002) derived an inventory model for linear trend in demand. Other researchers such as Silver and Peterson (1985), Roy, Kar, Maiti, and Maiti (2009), and Goyal and Giri (2003) studied an inventory model with time varying demand. Karimi-Nasab, Dowlatshahi, and Heidari (2013) developed a multiobjective distribution-pricing model for multiperiod price-sensitive demands. In the literature there are many papers on demand. Recently, in the competitive market on a long run business system, it is seen that the demand faces a competition and sale is destroyed by depreciation. In such circumstances, the advertisement policy has a positive effect to the demand rate. The advertisement through IT by electronic media, print media, etc. improves the communication between customers and companies and it helps us to give the accurate information flow to the customer about the product of the companies. So in the present era of information technology (IT), the advertisement plays an important role to control the demand of the product in the market. For this reason, Cho (1996) developed an optimal production and advertising policies in crisp environment. Recently, Hazari, Maity, Dey, and Kar (2015) developed an imperfect production inventory model in bi-fuzzy environment with the same idea.

This paper extends the traditional EPQ model by accounting an imperfect production inventory model with advertisement dependent demand rate in which the advertisement rate is increasing with time at a decreasing rate to recover the sale which is destroyed by depreciation and it increases the acceptability of the product in the competition market. Here, the production rate is considered as a decision variable. This paper also considers the issue that the imperfect items are sold as a single batch at the end of 100% screening process at a reduced price. The screening rate has been considered to be different with the production rate and it has been varied proportionally to the production rate. In this paper, a new type of defective rate has been considered depending upon the production rate. Under these considerations, a mathematical model has been developed to get the maximum profit from the system. Finally, some examples have been provided to illustrate the feasibility of the model numerically. The detailed comparative statement of the proposed model with the existing literature has been given in Table 1.

The remainder of this paper is organized as follows: Section 2 contains notations and assumptions. Section 3 is for mathematical formulation of the proposed imperfect production inventory model. Section 4 is for solution methodology. Section 5 provides some numerical examples. Section 6 is for sensitivity analysis. Finally, the paper is concluded including the future research in Section 7.

2. Notations and assumptions

The following notations and assumptions have been used for developing the proposed model:

2.1. Notations

In this subsection, we present the notations used in the proposed model. The following parameters and decision variables are used in the problem formulation.

Parameters	
$q(t)$	on hand inventory of produced item (perfect and imperfect quality) in the production center
$q_1(t)$	on hand inventory of perfect quality item after screening the produced item
$q_2(t)$	on hand inventory of imperfect quality item after screening the produced item
Q_0	minimum stock level maintained for perfect quality item
$D(t)$	demand rate of the market
x	percentage of screening item in the screening center per unit time
η	depreciation rate
$v(t)$	advertisement rate
c_p	production cost per unit item
c_{sr}	screening cost per unit item
h_c	inventory holding cost per unit time per unit item in production center
h_m	inventory holding cost per unit time per unit item of perfect quality
h'_m	inventory holding cost per unit time per unit item of imperfect quality
s	selling price per unit for perfect quality item
s'	selling price per unit for imperfect quality item

Decision variables	
P	production rate of the manufacturer ($P \geqslant D(t)$)
T	total time-length of the business period

Related to the decision variables of P or/and T	
t_1	duration of production run time
t_2	duration of screening run time
θ	defective production rate, i.e., the rate of producing defective units

2.2. Assumptions

In this subsection, the most fundamental assumptions of the problem are summarized as follows.

(i) It is a single item production inventory model in infinite time horizon.

(ii) Here it is assumed that the manufacturer has capability to collect sufficient raw materials, labors, machines and other related resources to produce the item. So he/she wants to produce the optimum amount of item per unit time to get the maximum profit from his/her business. In this sense, here the production rate (P) has been assumed to be a variable.

(iii) It is well known that the quality of a product depends on raw material, labor experience, machine component, production rate, etc. Since here the production rate has been considered as a variable, hence the defective rate of the produced items must be dependent on the production rate. For this reason, the defective production rate (θ) has been considered as follows:

$$\theta = \theta_0 - \frac{\theta_1}{P} \tag{1}$$

where θ_0 and θ_1 be the positive constants.