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# Monte Carlo simulation analysis of the effects of different system performance levels on the importance of multi-state components

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**Abstract**

In the present paper, we consider some frequently used importance measures, in their generalized form proposed by the authors for application to multi-state systems constituted by multi-state components. To catch the dynamics of multi-state systems, Monte Carlo simulation has been utilized. A simulation approach has been presented which allows estimating of all the importance measures of the components at a given performance level in a single simulation, provided that the components are independent. The effects of different performance demands made on the system on the importance of its multi-state components have been examined with respect to a simple multi-state series–parallel system. The results have shown that a performance level of a component may be more critical for the achievement of a system performance and less critical for another.


*Keywords:* Multi-state system; Monte Carlo simulation; Importance measures

## 1. Introduction

Information about the importance of the components constituting a system, with respect to its safety, reliability and availability, is of great practical aid to system designers and managers. Indeed, the identification of which components mostly determine the overall system behaviour allows one to trace system bottlenecks and provides guidelines for effective actions of system improvement.

For this purpose, Importance Measures (IMs) were first introduced by Birnbaum [1]. The Birnbaum measure deals with the effects of changes in the unreliability of a given component. Components for which a variation of the unreliability results in the largest variation of the system unreliability have the highest Birnbaum importance. Fussell and Vesely [2] later proposed a measure based on the cut-sets importance. According to the Fussell–Vesely measure, the importance of a component depends on the number and on the order of the cut-sets in which it appears. Other concepts of IMs have been proposed and used, based on different views of the influence of the components on the system performance.

Recently, IMs are being widely used in risk-informed applications to characterize the importance of basic events, i.e. component failures, human errors, common cause failures, etc. with respect to the risk associated to the system [3–6]. In this framework, two other measures are frequently used: the risk reduction worth and the risk achievement worth [3]. The former one is a measure of the 'worth' of the basic event in achieving the present level of system risk and, when applied to components, it highlights the importance of maintaining the current level of reliability with respect to the basic failure event associated to such components. The latter one, the risk reduction worth, is associated with the maximum decrease in risk consequent to an improvement of the element associated with the basic event considered.

The above mentioned measures have been applied to systems made up of binary components (i.e. components which can be in two states: functioning or faulty). This hypothesis does not fit with the real functioning of many systems, such as; for example, manufacturing production, power generation and transportation systems. For such systems, an overall performance is defined, which can settle on different levels (e.g. 100, 80, 50% of the nominal capacity), depending on the operative conditions of the constitutive multi-state components (e.g. a control valve

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**Nomenclature**

| | |
|---|---|
| $\tau$ | mission time |
| $n$ | number of components constituting the system |
| $m_i$ | number of states of component $i$, $i = 1, 2, \ldots, n$ |
| $m_{sys} = \prod_{i=1}^{n} m_i$ | number of system configurations or states |
| $\eta$ | number of series nodes (macro-components) |
| $n_l$ | number of parallel branches of node $l$, $l = 1, 2, \ldots, \eta$ |
| $\lambda_{i,j\to k}$ ($h^{-1}$) | transition rate of component $i$ from state $j$ to state $k$, $j$, $k = 1, 2, \ldots, m_i$ |
| $\Lambda^i$ | transition matrix of component $i$ |
| $p_{i,j}(t)$ | probability of component $i$ being in state $j$ at time $t$ |
| $\bar{p}_{i,j}$ | mean probability of component $i$ being in state $j$ over the mission time $\tau$ |
| $w_{i,j}$ | performance of component $i$ when operating in state $j$ |
| $\bar{w}_i$ | mean performance of component $i$ over the mission time $\tau$ |
| $\alpha$ | generic level of component performance |
| $\Gamma_i^\alpha$ | set of states of component $i$ characterized by a performance level below or equal to $\alpha$ |
| $\bar{\Gamma}_i^\alpha$ | set of states of component $i$ characterized by a performance level above $\alpha$ (complement set of $\Gamma_i^\alpha$) |
| $\underline{j} = (j_1, j_2, \ldots, j_n)$ | system state; $j_i$ represents the state of the $i$th component, $i = 1, 2, \ldots, n$ |
| $P_{\underline{j}}(t)$ | probability of the system being at time $t$ in state $\underline{j}$ |
| $W_{\underline{j}}$ | performance of the system when in state $\underline{j}$ |
| $W^*(t)$ | minimum level of system performance required at time $t$ |
| $A(W^*, t) = \sum_{W_{\underline{j}} \geq W^*(t)} P_{\underline{j}}(t)$ | MSS-availability, availability of the Multi-State System at time $t$ |
| $U(W^*, t) = 1 - A(W^*, t)$ | MSS-unavailability, unavailability of the Multi-State System at time $t$ |
| $\bar{U}(W^*)$ | mean MSS-unavailability over the mission time $\tau$ |
| $\bar{U}_i^{\leq\alpha}(W^*) = \bar{U}(W^* \mid j_i \in \Gamma_i^\alpha \text{ in } [0, \tau])$ | system mean MSS-unavailability when the performance of the $i$th component is restricted to be below or equal to $\alpha$ (i.e. $j_i \in \Gamma_i^\alpha$) in $t \in [0, \tau]$ |
| $\bar{U}_i^{>\alpha}(W^*) = \bar{U}(W^* \mid j_i \in \bar{\Gamma}_i^\alpha \text{ in } [0, \tau])$ | system mean MSS-unavailability when the performance of the $i$th component is restricted to be above $\alpha$ (i.e. $j_i \in \bar{\Gamma}_i^\alpha$) in $[0, \tau]$ |
| $i^{\leq\alpha}$ | pseudo-component associated to component $i$ restricted to evolve among states $j_i \in \Gamma_i^\alpha$ |
| $i^{>\alpha}$ | pseudo-component associated to component $i$ restricted to evolve among states $j_i \in \bar{\Gamma}_i^\alpha$ |
| $p_{i^{\leq\alpha},j}(t)$ | probability of the pseudo-component $i^{\leq\alpha}$ being in state $j$ at time $t$ |
| $p_{i^{>\alpha},j}(t)$ | probability of the pseudo-component $i^{>\alpha}$ being in state $j$ at time $t$ |
| $\underline{j}^{\leq\alpha} = (j_{1^{\leq\alpha}}, j_{2^{\leq\alpha}}, \ldots, j_{n^{\leq\alpha}})$ | state vector of the system constituted by components $i^{\leq\alpha}$, $i = 1, 2, \ldots, n$; $j_{i^{\leq\alpha}}$ represents the state of component $i^{\leq\alpha}$ |
| $\underline{j}^{>\alpha} = (j_{1^{>\alpha}}, j_{2^{>\alpha}}, \ldots, j_{n^{>\alpha}})$ | state vector of the system constituted by components $i^{>\alpha}$, $i = 1, 2, \ldots, n$; $j_{i^{>\alpha}}$ represents the state of component $i^{>\alpha}$ |
| $\underline{j}^* = (\underline{j}, \underline{j}^{\leq\alpha}, \underline{j}^{>\alpha})$ | state vector of the extended system constituted by components $i, i^{\leq\alpha}, i^{>\alpha}, i = 1, 2, \ldots, n$ |
| $\underline{j}_{i^{\leq\alpha}} = (j_1, j_2, \ldots, j_{i^{\leq\alpha}}, \ldots, j_n)$ | state vector of the system with the generic $i$th component replaced by $i^{\leq\alpha}$, $i = 1, 2, \ldots, n$ |
| $\underline{j}_{i^{>\alpha}} = (j_1, j_2, \ldots j_{i^{>\alpha}}, \ldots, j_n)$ | state vector of the system with the generic $i$th component replaced by $i^{>\alpha}$, $i = 1, 2, \ldots, n$ |
| $t_k$ | time of occurrence of the $k$th stochastic transition of the considered system |
| $\underline{j}_k$, $\underline{j}_k^{\leq\alpha}$, $\underline{j}_k^{>\alpha}$, $\underline{j}_k^*$, $\underline{j}_{i^{\leq\alpha},k}$, $\underline{j}_{i^{>\alpha},k}$ | states of the various systems entered at time $t_k$ |
| $ua_i^{MSS,\alpha}$ | MSS-unavailability achievement worth of $\alpha$-level of component $i$ |
| $ur_i^{MSS,\alpha}$ | MSS-unavailability reduction worth of $\alpha$-level of component $i$ |
| $uFV_i^{MSS,\alpha}$ | Fussell–Vesely MSS-unavailability measure of $\alpha$-level of component $i$ |
| $uB_i^{MSS,\alpha}$ | Birnbaum MSS-unavailability measure of $\alpha$-level of component $i$ |

**Acronyms**

| | |
|---|---|
| MC | Monte Carlo |
| MSS | multi-State-System |
| IMs | importance measures |
| $IM^{MSS}s$ | multi-state system importance measures |
| PIMs | performance importance measures |