A Branch and Bound Algorithm for Three-Machine Flow Shop with Overlapping Waiting Time Constraints

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Abstract: This paper focuses on a three-machine flow shop scheduling problem with overlapping waiting time constraints for makespan minimization. In the problem, waiting times of each job between the first two machines and between the first and third machines are constrained by two distinctive time limits, respectively. Such overlapping waiting time constraints are one of the common scheduling requirements in semiconductor manufacturing since up to 20% of whole process steps are controlled with waiting time limits for better quality. For example, wafers completed with chemical treatment on a diffusion machine must be cleaned on a cleaning machine and then should be processed on another diffusion machine. In this case, two waiting time limits are independently applied between diffusion and cleaning machines and between two diffusion machines, respectively. We first identify several dominance properties of the problem and develop a branch and bound algorithm using the properties. We use heuristic algorithms to obtain a good initial solution and derive five different lower bounds. Computational tests are performed for evaluating the performance of the algorithm.

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1. INTRODUCTION

We consider a three-machine flow shop scheduling problem with overlapping waiting time constraints for makespan minimization. In the problem under consideration, waiting times of job $i$ between the first two machines and between the first and third machines are constrained by two distinctive time limits, $w_{1i}$ and $w_{2i}$, respectively. The problem can be denoted by $F_3|w_{1}, w_{2}|C_{\text{max}}$ in the three-field notation of Graham et al. (1979) where $C_{\text{max}}$ is the makespan. Jobs should be processed on the second and third machines within certain periods of time after those jobs are completed on the first machine. Such overlapping waiting time constraints are one of the common scheduling requirements in semiconductor manufacturing since up to 20% of whole process steps are controlled with waiting time limits for better quality. For example, wafers completed with chemical treatment on a diffusion machine must be cleaned on a cleaning machine and then should be processed on another diffusion machine. In this case, two waiting time limits are independently applied between diffusion and cleaning machines and between two diffusion machines, respectively. If the wafers cannot be started within the time period, they are abandoned or reprocessed (Joo and Kim, 2009).

In $m$-machine flow shop, all of $n$ jobs are processed on machine $j$ for $j = 1, 2, ..., m$ in sequence. Flow shop scheduling problems have been studied extensively (Framinan et al., 2004; Hejazi and Saghafian, 2005; Gupta and Stafford, 2006). However, there are not many papers on flow shop with waiting time constraints or limited waiting time. Yang and Chern (1995) have addressed a two-machine flow shop problem with limited waiting time constraints for makespan minimization. They have shown that the problem is NP-hard and proposed a branch and bound (B&B) algorithm. Fondrevelle et al. (2006) have also dealt with the same problem for $m$-machine flow shop in which minimal and maximal time lags between every pair of successive operations are required. Joo and Kim (2009) have examined a two-machine flow shop problem with limited waiting time constraints and developed many dominance properties used in a B&B algorithm. An et al. (2016) have further analyzed the problem with sequence-dependent setup times. There have also been many studies on flow shop scheduling with material handling systems (Kim et al., 2013, 2015; Lee et al., 2014; Lee and Kim, 2016). Most studies on developing optimal schedules of flow shop with waiting time constraints are restricted to a two-machine problem. In addition, there is no study considering overlapping waiting time constraints.

In this paper, we propose a B&B algorithm for a three-machine flow shop scheduling problem with overlapping waiting time constraints for the objective of minimizing makespan. This problem is easily proven to be NP-hard since three-machine no-wait flow shop is NP-hard (Rock, 2009).
Fig. 1. Problem description

1984), which is a special case of our problem. We develop dominance properties and lower bounds to reduce the search space of the B&B tree. We also use heuristic algorithms for an initial upper bound. We first describe the problem in detail.

2. PROBLEM DESCRIPTION

In our problem, there are \( n \) jobs to be processed on three machines in the order of machines 1, 2 and 3. All jobs are available at the beginning and their processing times are deterministic and known. Setup times which are sequence-independent are assumed to be included in processing times. Once a job is processed on the first machine, it must start processing on the second and third machines within certain periods of time. Waiting times of jobs are all different. Each machine can process one job at a time, and there is one machine in each stage. Fig. 1(a) and (b) show a brief description of the problem with overlapping waiting time constraints. Each job \( i \) has two distinctive time constraints, \( w_{i1} \) and \( w_{i2} \), between the first and second machines and between the first and third machines, respectively, as in Fig. 1(a).

We assume permutation schedules in which the sequence of jobs on the three machines is the same. Even though permutation schedules may not be dominant with waiting time constraints, they are often used in industry due to their simplicity and technical restrictions on material handling systems (An et al., 2016). We note that permutation schedules are dominant in three-machine flow shop without waiting time constraints. In this paper, we use the following notation:

- \( i, j \): indices of jobs
- \( k \): index of machines \( k = 1, 2, 3 \)
- \( r \): index of a job at the \( r \)th position in a (partial) schedule
- \( p_{ik} \): processing time of job \( i \) on machine \( k \)
- \( w_{i1} \): waiting time of job \( i \) between the first and second machines
- \( w_{i2} \): waiting time of job \( i \) between the first and third machines
- \( \sigma \): partial schedule (sequence)
- \( \sigma_{ij...m} \): partial schedule composed of \( \sigma \) followed by jobs \( i, j, ..., m \) in this order

\( S_{[r]k} \): start time of the \( r \)th job on machine \( k \)
\( C_{[r]k} \): completion time of a job at the \( r \)th position on machine \( k \)
\( C_k(\sigma) \): completion time of (all jobs in) partial schedule \( \sigma \) on machine \( k \)

We note that \( w_{i2} \) does not include \( p_{i2} \). With the notation, completion times of jobs in a given sequence can be obtained as follows:

\[
C_{[r]1} = p_{[r]1},
\]
\[
C_{[r]2} = p_{[r]1} + p_{[r]2},
\]
\[
C_{[r]3} = p_{[r]1} + p_{[r]2} + p_{[r]3},
\]
\[
C_{i} = \max\{C_{[r-i]1} + p_{[r]1}, C_{[r-i]2} - w_{[i]1}, C_{[r-i]3} - w_{[i]2} - p_{[r]2}\} \text{ for } i = 2, ..., n.
\]
\[
C_{i} = \max\{C_{[r-i]2}, C_{[r]1}\} + p_{[r]2} \text{ for } i = 2, ..., n.
\]
\[
C_{i} = \max\{C_{[r-i]3}, C_{[r]2}\} + p_{[r]3} \text{ for } i = 2, ..., n.
\]

We now provide a mathematical formulation with the following decision variables.

\[
x_{ir} = 1 \text{ if job } i \text{ is assigned to the } r \text{th position in a (partial) schedule, and 0 otherwise.}
\]
\[
y_{ijr} = 1 \text{ if job } i \text{ is assigned to the } r \text{th position and job } j \text{ is assigned to the } (r + 1)\text{th position, and 0 otherwise.}
\]

Minimize \( C_{\text{max}} \) (1)

Subject to
\[
\sum_{i=1}^{n} x_{ir} = 1, \quad r = 1, 2, ..., n \quad (2)
\]
\[
\sum_{r=1}^{n} x_{ir} = 1, \quad i = 1, 2, ..., n \quad (3)
\]
\[
S_{[r]1} + \sum_{i=1}^{n} p_{i1} x_{ir} \leq S_{[r+1]1}, \quad r = 1, 2, ..., n - 1 \quad (4)
\]
\[
S_{[r]2} + \sum_{i=1}^{n} p_{i2} x_{ir} \leq S_{[r+1]2}, \quad r = 1, 2, ..., n - 1 \quad (5)
\]
\[
S_{[r]3} + \sum_{i=1}^{n} p_{i3} x_{ir} \leq S_{[r+1]3}, \quad r = 1, 2, ..., n - 1 \quad (6)
\]
\[
S_{[r]1} + \sum_{i=1}^{n} p_{i1} x_{ir} \leq S_{[r]2}, \quad r = 1, 2, ..., n \quad (7)
\]
\[
S_{[r]2} + \sum_{i=1}^{n} p_{i1} x_{ir} \leq S_{[r]3}, \quad r = 1, 2, ..., n \quad (8)
\]
\[
S_{[r]1} + \sum_{i=1}^{n} (p_{i1} + w_{i1}) x_{ir} \geq S_{[r]2}, \quad r = 1, 2, ..., n \quad (9)
\]
\[
S_{[r]1} + \sum_{i=1}^{n} (p_{i1} + p_{i2} + w_{i2}) x_{ir} \geq S_{[r]3}, \quad r = 1, 2, ..., n \quad (10)
\]
\[
x_{ir} + x_{j(r+1)} - 1 \leq y_{ijr}, \quad \forall i \neq j, r = 1, 2, ..., n \quad (11)
\]
\[
y_{ijr} \leq x_{ir} \quad \forall i \neq j, r = 1, 2, ..., n - 1 \quad (12)
\]
\[
y_{ijr} \leq x_{j(r+1)} \quad \forall i \neq j, r = 1, 2, ..., n - 1 \quad (13)
\]
\[
C_{\text{max}} \geq S_{[n]3} + \sum_{i=1}^{n} p_{i3} x_{in} \quad (14)
\]
\[
x_{ir}, y_{ijr} \in \{0, 1\} \quad \forall i \neq j \quad \forall r \quad (15)
\]
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