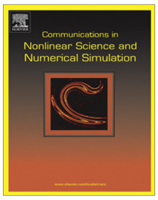
Commun Nonlinear Sci Numer Simulat 19 (2014) 298–310

Contents lists available at SciVerse ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns



Numerical optimization of tuned mass absorbers attached to strongly nonlinear Duffing oscillator

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ARTICLE INFO

Article history:
Received 4 February 2013
Received in revised form 3 June 2013
Accepted 3 June 2013
Available online 13 June 2013

Keywords:
Duffing oscillator
Tuned mass absorber
Energy absorption
Optimization

ABSTRACT

We investigate the dynamics of the vertically forced Duffing oscillator with suspended tuned mass absorber. Three different types of tuned mass absorbers are taken into consideration, i.e., classical single pendulum, dual pendulum and pendulum-spring. We numerically adjust parameters of absorbers to obtain the best damping properties with the lowest mass of attached system. The modification of classical case (single pendulum) gives the decrease of Duffing system amplitude. We present strategy of parameters tuning which can be easily applied in a large class of systems.

1. Introduction

A tuned mass damper (TMD) was patented by Frahm in 1909 [1]. His device was a linear oscillator which consists of mass and linear spring with the same natural frequency as a damped system. Under this condition one can avoid resonance of main mass, but the decrease of amplitude is observed only around resonant frequency. The next important modification of the TMD was the addition of a dash-pot [2] which implies the increase of the range of frequencies for which effective suppressing of oscillations is observed. There are a lot of modifications of the classical passive TMD, most of them have important practical applications, i.e., to prevent damage of buildings due to seismic excitation [3,4], to suppress vibration of tall buildings subjected to wind [5,6], to achieve the best properties of cutting processes [7,8], to mitigate vibration of floors or balconies [9,10], to reach stable rotations of rotors [11–13], or to stabilize drill strings [14] and many more. Despite the fact that scientists and engineers are working on designing the best passive device, there are also lots of efforts to improve properties of TMD by adding control (hybrid, semi-active and active systems) [15–19].

The linear TMD decreases oscillations of the main system only around its resonant frequency (also natural frequency of the TMD), but outside this range one can observe an increase of amplitude. The solution of this problem was proposed by Roberstson [20] and Arnold [21]. They replaced linear spring of the TMD by the nonlinear one (with linear and nonlinear parts of stiffness). This resulted in the improvement of damping properties when compared with the classic design. In recent years much more attention was paid to the possibility of purely nonlinear spring implementation [22–24]. The authors show that with such a spring there is no prominent damped frequency and the TMD works in wide range of excitation frequencies.

Simultaneously with improving the TMD Hatwal et al. [25–27] proposed the device called a tuned mass absorber (TMA) where the linear (nonlinear) oscillator is replaced by a pendulum. As the natural frequency of a pendulum depends only on its length, it is much easier to tune it in practical applications. The pendulum is used as the TMA independently on the excitation direction, in horizontal case the pendulum is oscillating for any frequency while for vertical direction only in

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http://dx.doi.org/10.1016/j.cnsns.2013.06.001

its parametric resonances. The dynamics of the TMA with vertical forcing of the base mass was considered in a few papers [26–37]. Presented analysis allows to understand the dynamics and the response of the main mass around primary and secondary resonances of the pendulum. In our previous paper [38] we presented complete bifurcation analysis of the TMA applied to forced Duffing oscillator in two parameters space (the amplitude and the frequency of excitation). We showed oscillatory and rotational periodic solutions (internal resonances) and their coexistence. The same phenomena were also observed for systems where main mass is oscillating horizontally [39–42] and for combined veritcal and horizontal excitation [43].

The recent important studies on the TMD and the TMA take into account devices that consist of many single systems or with more than one degree of freedom. In [42,44–46] one can find an application of multiple TMD with natural frequencies distributed over a defined range of frequencies. Such a construction damps the motion of the primary system more effectively than single TMD. Another advantage of multiple TMD is the reduction of the mass of individual TMD. Alternative construction of multiple TMD is connecting them in series: linear oscillators [47], linear and nonlinear systems [15] and purely nonlinear devices [48]. All three approaches give better damping properties than single TMD. There are also a lot of publications on the multiple TMA. Starting from works of Vyas and Bajaj [49,50], where authors increase efficiency of TMA by differentiation pendulums lengths. Significant advantages of this set up was confirmed experimentally by Ikeda [35]. As in the case of TMD one can find many different construction of multidegree TMA, i.e., rotational pendulums TMA [51,52] or the TMA with rotational and translational movements [53]. The connection of pendulums in series (double pendulum) is efficient [54] but causes a lot of practical problems – its dynamics is very complex and one can not be sure that desired attractor will be achieved [55,56].

In this paper we consider three different types of TMA suspended on the forced Duffing oscillator. The purpose of our analysis is to study and compare energy absorption properties of each system. We show that by careful choice of parameters one can achieve large decrease of Duffing system amplitude.

In Section 2 we show models of systems under consideration. Section 3 is devoted to optimization of single TMA parameters. In Sections 4 and 5 we show how modifications of classical TMA influence damping efficiency. Finally, in Section 6 we conclude on our investigations.

2. Model of the system

The horizontally forced single-well Duffing oscillator, which we consider here as a base system, is shown in Fig. 1(a). In Fig. 1(b) one can see a classical TMA with single pendulum mounted on the Duffing system. The first modification assumes

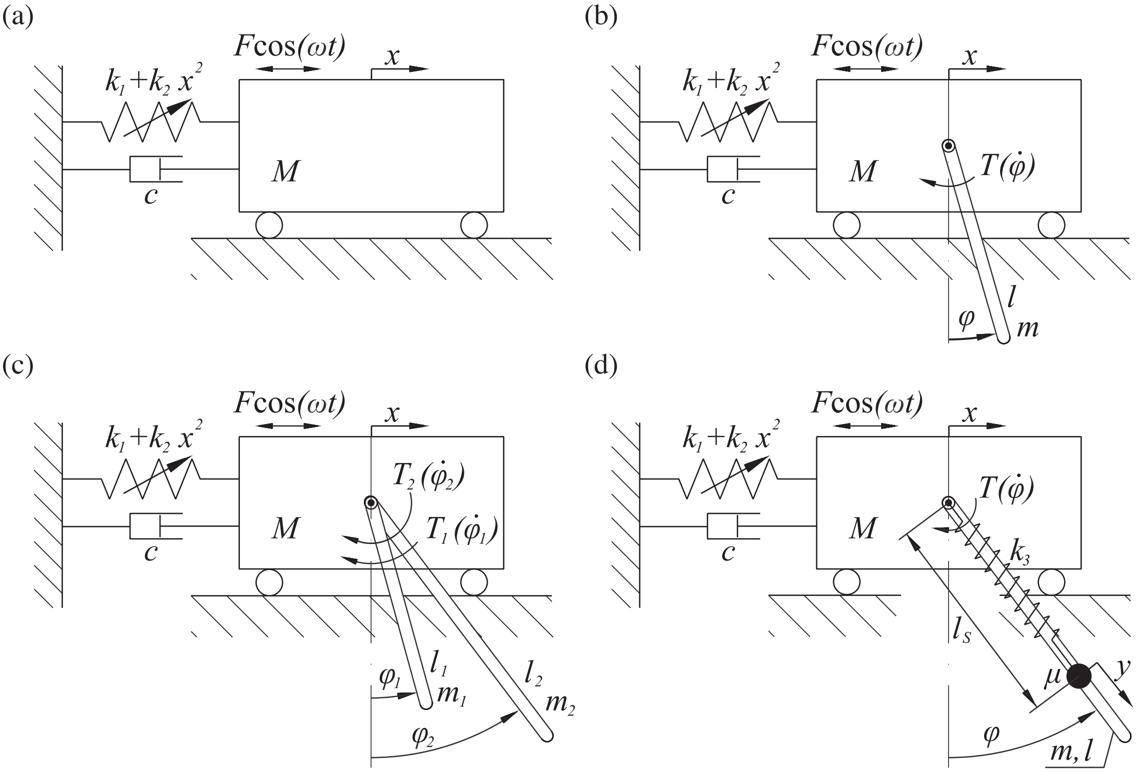

Fig. 1. Model of Duffing oscillator (a); classical TMA (b); dual-pendulum TMA (c); pendulum-spring TMA (d).