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A scalable variational inequality approach for flow through porous media models with pressure-dependent viscosity

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ABSTRACT

Mathematical models for flow through porous media typically enjoy the so-called maximum principles, which place bounds on the pressure field. It is highly desirable to preserve these bounds on the pressure field in predictive numerical simulations, that is, one needs to satisfy discrete maximum principles (DMP). Unfortunately, many of the existing formulations for flow through porous media models do *not* satisfy DMP. This paper presents a robust, scalable numerical formulation based on variational inequalities (VI), to model non-linear flows through heterogeneous, anisotropic porous media without violating DMP. VI is an optimization technique that places bounds on the numerical solutions of partial differential equations. To crystallize the ideas, a modification to Darcy equations by taking into account pressure-dependent viscosity will be discretized using the lowest-order Raviart–Thomas (RT0) and Variational Multi-scale (VMS) finite element formulations. It will be shown that these formulations violate DMP, and, in fact, these violations increase with an increase in anisotropy. It will be shown that the proposed VI-based formulation provides a viable route to enforce DMP. Moreover, it will be shown that the proposed formulation is scalable, and can work with any numerical discretization and weak form. A series of numerical benchmark problems are solved to demonstrate the effects of heterogeneity, anisotropy and non-linearity on DMP violations under the two chosen formulations (RT0 and VMS), and that of non-linearity on solver convergence for the proposed VI-based formulation. Parallel scalability on modern computational platforms will be illustrated through strong-scaling studies, which will prove the efficiency of the proposed formulation in a parallel setting. Algorithmic scalability as the problem size is scaled up will be demonstrated through novel static-scaling studies. The performed static-scaling studies can serve as a guide for users to be able to select an appropriate discretization for a given problem size.

1. Introduction

The success of many current and emerging technological endeavors critically depend on a firm understanding and on the ability to control flows in heterogeneous, anisotropic porous media. These endeavors include geological carbon sequestration,

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geothermal systems, oil recovery, water purification systems, extraction of gas hydrates from tight shale; just to name a few. Modeling and predictive simulations play an important role in all these endeavors, and one has to overcome many numerical challenges to obtain accurate numerical solutions. It is beyond the scope of this paper to address all the major issues associated with the flow of fluids through porous media. Herein, we however address one of the main numerical challenges that is encountered in numerical modeling of flow through porous media with relevance to the mentioned applications.

Flow through porous media models typically enjoy the so-called maximum principles, which place bounds on the pressure field. These bounds depend on the prescribed data, which include boundary conditions, anisotropy of the porous media, body force, volumetric source, topology of the domain, and the regularity of the boundary. The non-negative constraint on the pressure (which basically implies the physical condition that a fluid subject to a flow in a porous medium cannot sustain a "suction" by itself) can be shown to be a special case of the classical maximum principle. It is imperative that these bounds on the pressure field are preserved in a predictive numerical simulation; that is, one needs to satisfy maximum principles in the discrete setting. The discrete version of maximum principles is commonly referred to as discrete maximum principles (DMP). It becomes even more crucial for those flow models in which the material properties depend on the pressure; for example, the case in which the viscosity of the fluid depends on the pressure in the fluid, as a violation of DMP can amplify errors in the solution fields. Unfortunately, many of the commonly used mixed finite element formulations for flow through porous media models do *not* satisfy DMP, which will be shown in the subsequent sections. Moreover, the problems pertaining to flow through porous media, especially the ones encountered in subsurface modeling, are highly nonlinear and large-scale in nature. Thus, one needs to develop numerical formulations that are scalable in an algorithmic and parallel sense in addition to satisfying DMP.

This paper presents a new, scalable numerical formulation based on variational inequalities (VI) that enforces discrete maximum principles for nonlinear flow through porous media models by taking into account heterogeneity, anisotropic permeability and pressure-dependent viscosity.

1.1. A review of related prior works

In order to bring out clearly the contributions made in this paper and the approach taken by us, we provide a brief discussion on prior works with respect to three aspects.

1.1.1. Pressure-dependent viscosity

The classical Darcy model [19], which is the most popular flow through porous media model, assumes the viscosity of the fluid to be a constant, and in particular, the model assumes that the coefficient of viscosity is independent of the pressure in the fluid [46]. But there is abundant experimental evidence that the viscosity of liquids, especially organic liquids, depends on the pressure [10]. More importantly, the dependence of viscosity on pressure for organic liquids is exponential [3]. Since then several studies have developed mathematical models that take into account the dependence of viscosity on pressure, and established the existence of solutions for the resulting governing equations [39,30,24,11]. A work that is relevant to this paper is by [46] who derived a modification to the Darcy model using the mixture theory by taking into account the pressure-dependent viscosity. They have also developed a stabilized formulation for the resulting equations using the variational multiscale paradigm [32], and have shown, using numerical simulations, that the dependence of viscosity on pressure has a significant effect on both qualitative and quantitative nature of the solution fields. Later, [47] have presented a stabilized mixed formulation based on Picard linearization and laid down the differences in the predictions for enhanced oil recovery and carbon sequestration when the pressure dependence on viscosity is considered against when not considered. Recently, [15] have extended the pressure dependence to the Darcy–Forchheimer model, and demonstrated how the dependence of the drag coefficient on pressure differs significantly from when it depends on velocity. *However, all of these studies considered isotropic permeability, and did not address the violations of maximum principles and the non-negative constraint on the pressure field.*

1.1.2. Anisotropy, violations of DMP and numerical techniques to enforce DMP

Varga [59] was the first to address DMP, and the study was restricted to the finite difference method applied on the Poisson's equation. Ciarlet and Raviart [17] were the first to address DMP in the context of the finite element method. Their study revealed that the single-field Galerkin formulation for solving the Poisson's equation, in general, does not satisfy DMP. They also obtained sufficient conditions which are in the form of restrictions on the mesh (e.g., all the angles of a triangular element to be acute) to meet DMP. Subsequent studies have found that these mesh restrictions, which have been derived for isotropic diffusion equations, are not sufficient when one considers anisotropic diffusion equations or other processes like advection and reactions; for example, see [41] and references therein.

In the last decade, several approaches have been developed to enforce DMP on general computational grids for anisotropic diffusion-type equations under the finite element method. Some of the notable approaches are based on either constrained optimization techniques [35,45,42], placing anisotropic metric-based restrictions on the mesh [31,41], or altering the formulations at the continuum setting [51]. Placing restrictions on meshes is not a viable approach for applications involving flow through porous media, as the computational domains are complex and it is not practical or even possible to generate metric-based meshes that satisfy DMP. The approach of altering formulations at the continuum setting