Integrably bounded set-valued stochastic integrals

Michał Kisielewicz *, Mariusz Michta *

Faculty of Mathematics Computer Science and Econometrics, University of Zielona Góra, Podgórna 50,
65-246 Zielona Góra, Poland

A R T I C L E   I N F O

Article history:
Received 4 December 2014
Available online xxxx
Submitted by V. Pozdnyakov

Keywords:
Set-valued mapping
Set-valued integral
Set-valued stochastic process

A B S T R A C T

The paper is devoted to properties of Aumann and Itô set-valued stochastic integrals, defined as some set-valued random variables. In particular the problem of integrable boundedness of the generalized Itô set-valued stochastic integrals is considered. Unfortunately, Itô set-valued stochastic integrals, defined by E.J. Jung and J.H. Kim in the paper [5], are not in general integrably bounded (see [8,15]). Therefore, in the present paper we consider generalized Itô set-valued stochastic integrals (see [10,11]) defined for absolutely summable and countable subsets of the space \( L^2(\mathbb{R}^+ \times \Omega, \Sigma_F, \mathbb{R}^{d \times m}) \) of all square integrable IF-nonanticipative matrix-valued stochastic processes. Such integrals are integrably bounded and possess properties needed in the theory of set-valued stochastic equations.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

The paper deals with properties of Aumann and Itô set-valued stochastic integrals, defined as some set-valued random variables. Initial studies on Itô set-valued stochastic integrals, defined as subsets of the spaces \( L^2(\Omega, \mathbb{R}^n) \) and \( L^2(\Omega, \mathcal{X}) \), have been considered by F. Hiai and M. Kisielewicz (see [2,6,7]), where \( \mathcal{X} \) is a Hilbert space. Unfortunately, such defined integrals do not admit their representations by set-valued random variables with values in \( \mathbb{R}^n \) and \( \mathcal{X} \), because they are not decomposable subset of \( L^2(\Omega, \mathbb{R}^n) \) and \( L^2(\Omega, \mathcal{X}) \), respectively. J. Jung and J.H. Kim in [5] have defined the Itô set-valued stochastic integral as a set-valued random variable determined by a closed and decomposable hull of the set-valued stochastic functional integral defined in [6]. Unfortunately, such integrals are not in the general case (see [8,15]) integrably bounded. Therefore, in what follows we shall consider generalized Itô set-valued stochastic integrals (see [10,11]) of absolutely summable countable subsets of the space \( L^2(\mathbb{R}^+ \times \Omega, \Sigma_F, \mathbb{R}^{d \times m}) \) of square integrable IF-nonanticipative matrix-valued stochastic processes defined on a complete filtered probability space \( \mathcal{F} = (\Omega, \mathcal{F}, \mathbb{F}, \mathcal{P}) \). Generalized set-valued stochastic integrals were defined in the paper [11] and some of their properties have been considered in [10]. Let us recall (see [10] and [11]) that

* Corresponding authors.

E-mail addresses: M.Kisielewicz@wmie.uz.zgora.pl (M. Kisielewicz), M.Michta@wmie.uz.zgora.pl (M. Michta).

http://dx.doi.org/10.1016/j.jmaa.2017.01.013
0022-247X/© 2017 Elsevier Inc. All rights reserved.

for a given $m$-dimensional IF-Brownian motion $B = (B_t)_{t \geq 0}$ defined on a filtered probability space $(\mathcal{F}_t)_{t \geq 0}$ and a nonempty subset $\mathcal{G}$ of the space $L^2(\mathbb{R}^+ \times \Omega, \Sigma_{\mathbb{F}}, \mathbb{R}^{d \times m})$, a generalized Itô set-valued stochastic integral $\int_0^t G dB_t$ is understood as a $\mathcal{F}_t$-measurable set-valued random variable with values in the $d$-dimensional Euclidean space $\mathbb{R}^d$ and subtrajectory integrals $S_{\mathcal{F}}(\int_0^t G dB_t)$ are defined as equal to $\text{dec} J_t(G)$. Let $J_t$ denote the Itô isometry with values in the space $L^2(\Omega, \mathcal{F}_t, \mathbb{R}^{d \times m})$. In particular, if $\mathcal{G}$ is a nonempty decomposable subset of $L^2(\mathbb{R}^+ \times \Omega, \Sigma_{\mathbb{F}}, \mathbb{R}^{d \times m})$ then $\int_0^t G dB_t = \int_0^t G d\tau$, where $G = (G_t)_{t \geq 0}$ is an IF-nonanticipative set-valued process such that $S_{\mathbb{F}}(G) = \text{cl}_\mathcal{G}(G)$, where $S_{\mathbb{F}}(G) = \{g \in L^2(\mathbb{R}^+ \times \Omega, \Sigma_{\mathbb{F}}, \mathbb{R}^{d \times m}) : g_0(\omega) \in G_t(\omega) \}$ for a.e. $(t, \omega) \in \mathbb{R}^+ \times \Omega$.

In a similar way the Aumann set-valued stochastic integrals can be defined (see e.g. [9], p. 114 and [16]). Namely, for a given IF-nonanticipative set-valued stochastic process $F : \mathbb{R}^+ \times \Omega \to \text{Cl}(\mathbb{R}^d)$ such that $S_{\mathbb{F}}(F) \neq 0$, the Aumann set-valued stochastic integral $\int_0^t F d\tau$ is defined as the set-valued random variable such that $S_{\mathcal{F}}(\int_0^t F d\tau) = \text{dec} J_t(S_{\mathbb{F}}(F))$, where $J_t$ denotes for (fixed $t \geq 0$) the mapping with values in the space $L^2(\Omega, \mathcal{F}_t, \mathbb{R}^d)$ defined for every $f \in L^2(\mathbb{R}^+ \times \Omega, \Sigma_{\mathbb{F}}, \mathbb{R}^d)$ by setting $J_t(f) = \int_0^t f d\tau$.

Apart from the above mentioned Aumann set-valued stochastic integral $\int_0^t F d\tau$ one can define (see e.g. [9, 12] and [13]) an $\mathcal{A}$-set-valued stochastic integral $(\int_0^t F d\tau)$ by setting $((\mathcal{A}) \int_0^t F d\tau)(\omega) = \int_0^t F(\tau, \omega) d\tau$ for fixed $t \geq 0$ and a.e. $\omega \in \Omega$, where $\int_0^t F(\tau, \omega) d\tau$ denotes the parametrized Aumann integral, i.e., for a.e. fixed $\omega \in \Omega$ the Aumann integral of $F(\cdot, \omega)$. It can be verified (see [9], Lemma 3.1 of Chap. 3 and also [12, 13]) that $(\mathcal{A}) \int_0^t F d\tau$ is an $\mathcal{F}_t$-measurable compact valued set-valued random variable. For a.e. $\omega \in \Omega$ one has $(\mathcal{A}) \int_0^t F d\tau(\omega) = \{u(\omega) : u \in J_t(S_{\mathbb{F}}(F))\} \subset \{u(\omega) : u \in \text{dec} J_t(S_{\mathbb{F}}(F))\} \subset (\int_0^t F d\tau)(\omega)$.

It can be verified (see [9], Corollary 3.1 of Chap. 3) that for every measurable, convex-valued and integrably bounded set-valued process $F : \mathbb{R}^+ \times \Omega \to \text{Cl}(\mathbb{R}^d)$ one has $(\mathcal{A}) \int_0^t F d\tau(\omega) = (\int_0^t F d\tau)(\omega)$ for a.e. $\omega \in \Omega$ and $t \geq 0$. For other properties of $(\mathcal{A}) \int_0^t F d\tau$ we refer the reader to [9, 12] and [13].

Let us assume that $\mathcal{G} = \{g^n : n \geq 1\} \subset L^2(\mathbb{R}^+ \times \Omega, \Sigma_{\mathbb{F}}, \mathbb{R}^{d \times m})$ is absolutely summable, i.e. $\sum_{n=1}^\infty \|g^n\|^2 < \infty$, where $\|T\|_{\mathbb{F}}$ is a norm of $L^2(\mathbb{R}^+ \times \Omega, \Sigma_{\mathbb{F}}, \mathbb{R}^{d \times m})$. Then a generalized set-valued integral $\int_0^t G dB_t$ can be defined for every $t \geq 0$ as a set-valued random variable $H_t : \Omega \to \text{Cl}(\mathbb{R}^d)$ of the form $H_t(\omega) = \text{cl}_{\mathcal{G}}(\int_0^t g^n dB_t(\omega)) : n \geq 1$ for every $\omega \in \Omega$. Indeed, it is clear, that for every $t \geq 0$ a sequence $(\int_0^t g^n dB_t)_{n=1}^\infty$ is the Castaing representation of a set-valued random variable $H_t$. Furthermore we have $\sup_{n \geq 1} \mathbb{E} \|\int_0^t g^n dB_t\|^2_2 \leq \sum_{n=1}^\infty \|g^n\|^2 < \infty$. Then $(\int_0^t g^n dB_t)_{n=1}^\infty$ is a bounded sequence of $L^2(\Omega, \mathcal{F}_t, \mathbb{R}^d)$ contained in $S_{\mathcal{F}}(H_t)$. Therefore, by (9), Remark 3.6 of Chap. 2, we have $S_{\mathcal{F}}(H_t) = \text{dec} \sum_{n=1}^\infty \int_0^t g^n dB_t : n \geq 1 = \text{dec} \{J_t(g^n) : n \geq 1\} = \text{dec} J_t(G) = S_{\mathcal{F}}(\int_0^t G dB_t)$. Then $H_t = \int_0^t G dB_t$ a.s. for every $t \geq 0$.

Let us also note that for such a set $\mathcal{G} = \{g^n : n \geq 1\}$, a stochastic process $(\sum_{n=1}^\infty \|f^n dB_t\|^2_2)_{t \geq 0}$ of is a positive submartingale. Firstly, let us observe that such a set $\mathcal{G}$ is bounded in $L^2(\mathbb{R}^+ \times \Omega, \Sigma_{\mathbb{F}}, \mathbb{R}^{d \times m})$, because $\sup_{n \geq 1} \|g^n\|^2 \leq \sum_{n=1}^\infty \|g^n\|^2 < \infty$. Now, for a fixed $t \geq 0$, let us put $\xi^n = \sum_{k=1}^n \|f^n dB_t\|^2_2$ where $n \geq 1$ and $\xi^t = \sum_{n=1}^\infty \|f^n dB_t\|^2_2$. Let $m = \sum_{n=1}^\infty \|g^n\|^2$. We have $\xi^n \leq \xi^t$ a.s. and $E\xi^n \leq m < \infty$ for every $n \geq 1$ and $t \geq 0$. Therefore, $\sup_{n \geq 1} E\|\xi^n\|_{\mathbb{F}} < \infty$. The sequence $\xi^n_{t \geq 1}$ of positive random variables converges to $\xi^t$ a.s. for every fixed $t \geq 0$ and it is such that $\lim_n \infty E\xi^n_{\mathcal{F}_t} = E\xi^t_{\mathcal{F}_t}$ a.s. for every $0 \leq t < t < \infty$. On the other hand $\lim_{t \to \infty} E\xi^n_{\mathcal{F}_t} = \lim_n \infty \sum_{k=1}^n E\|f^n dB_t\|^2_2_F = \sum_{k=1}^\infty E\|f^n dB_t\|^2_2 F_a$ for every $0 \leq s < t < \infty$. Thus $E\sum_{n=1}^\infty \|f^n dB_t\|^2_2 F_a = \sum_{n=1}^\infty E\|f^n dB_t\|^2_2 F_a$ for every $0 \leq s < t < \infty$. Finally, by Jensen’s inequality we get $E\|f^n dB_t\|^2_2 F_a \geq \|f^n dB_t\|^2_2 F_a$ for every $n \geq 1$ and $0 \leq s < t < \infty$. Therefore, we have $\sum_{n=1}^\infty \|f^n dB_t\|^2_2 \leq \sum_{n=1}^\infty \|f^n dB_t\|^2_2 F_a$ a.s. for every $0 \leq s < t < \infty$.

Let $(X, \rho)$ be a metric space and denote by $C(X)$ a space of all nonempty closed subsets of $X$. For every $A, C \in C(X)$ let $\bar{h}(A, C) = \text{sup} \{d(a, C) : a \in A\}$, where $d(a, C) = \text{inf} \{\rho(a, c) : c \in C\}$. The Hausdorff distance $h(A, C)$ between $A, C \in C(X)$ is defined by $h(A, C) = \text{max} \{\bar{h}(A, C), \bar{h}(C, A)\}$. It can be verified (see [2], p. 24) that for every sequence $(A_n)_{n \geq 1} \subset C(X)$ converging in the Hausdorff metric...
دریافت فوری
متن کامل مقاله

امکان دانلود نسخه تمام متن مقالات انگلیسی
امکان دانلود نسخه ترجمه شده مقالات
پذیرش سفارش ترجمه تخصصی
امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
امکان دانلود رایگان ۲ صفحه اول هر مقاله
امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
دانلود فوری مقاله پس از پرداخت آنلاین
پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات