Periodic solution for the stochastic chemostat with general response function

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HIGHLIGHTS

- A stochastic chemostat model with periodic dilution rate and general response function is firstly proposed.
- Two kinds of sufficient conditions are given for the existence of the nontrivial positive periodic solution.
- The conditions for the existence of the periodic solution are more general than that in pre-existing papers.

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ABSTRACT

This paper addresses a stochastic chemostat model with periodic dilution rate and general class of response functions. The general functional response is assumed to satisfy two classifications of conditions, and these assumptions on the functional response are relative weak that are valid for many forms of growth response. For the chemostat with periodic dilution rate, we derive the sufficient criteria for the existence of the stochastic nontrivial positive periodic solution, by utilizing Khasminskii’s theory on periodic Markov process.

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1. Introduction

The chemostat is a basic piece of laboratory apparatus which consists of a series of vessels: the feed bottle, the culture vessel and the collecting vessel which are connected by pumps [1]. Chemostat is utilized for the continuous culture of microorganisms, and occupies a central place both in mathematical and theoretical ecology. In recent decades, modeling and researching of the chemostat has been an increasingly active field, and attracted great attention from mathematics and ecology [2–6]. One classic chemostat model with single species and single substrate that described by a system of ordinary differential equations can be written as the following form

\[
\begin{align*}
S'(t) &= D(S^0 - S(t)) - \frac{1}{\delta}p(S(t))x(t), \\
x'(t) &= -Dx(t) + p(S(t))x(t),
\end{align*}
\]  

(1.1)

where \(S(t), x(t)\) are the concentrations of nutrient and microbe at time \(t\), respectively; \(S^0\) is the original input concentration of nutrient and \(D\) represents the volumetric flow rate of the mixture of nutrient and microorganism, i.e. the common dilution

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rate. The term \( \frac{1}{\theta} p(S) \) denotes the uptake rate of substrate of the microbe population. We assume that \( p(S) \) represents the per-capita growth rate of the species and \( \delta \) is a growth yield constant. The growth response function \( p : \mathbb{R}^+ \to \mathbb{R}^+ \) is generally assumed to satisfy

\[
p \text{ is continuously differentiable},
\]

\[
p(0) = 0, \quad p(S) > 0 \quad \text{for} \quad S > 0.
\]

Butler et al. [7] investigated the global dynamics of a multiple competing species chemostat model with a general class of functions describing nutrient uptake. In the single-species case, we can derive from the study in [7] that there exists a uniquely defined positive real number \( 0 < \lambda \leq \infty \) such that \( p(S) < D \) for \( 0 < S < \lambda \); \( p(S) > D \) for \( S > \lambda \). Here \( \lambda \) represents the break-even concentration of the substrate for the species \( x(t) \). If \( \lambda < S^0 \), the solution of system (1.1) satisfies

\[
\lim_{t \to \infty} S(t) = \lambda, \quad \lim_{t \to \infty} x(t) = x^* = \delta(S^0 - \lambda),
\]

while when \( p(S^0) \leq D \), there exists a boundary equilibrium \( E_0 = (S^0, 0) \) which is asymptotically stable (\( E_0 \) is also called the washout equilibrium). In other words, in the case of any monotone uptake functions (such as the Monod functional response), when \( p(S^0) > D \), then the critical point \( E_0 = (\lambda, x^*) \) is globally asymptotically stable. Results in the case of multiple competing microbial populations are obtained in [8–10] which prove that the competitive exclusion principle holds in competition chemostat models.

In the traditional chemostat equations, two parameters are under the control of the experimenter, the concentration of the input nutrient \( S^0 \) and the dilution rate \( D \) (the pump rate). The chemostat can be applied in the waste water treatment process and industrial engineers, thus it is sensible to vary the dilution rate \( D \) with time. Moreover, biological populations are always subject to fluctuations that occur in a periodic phenomenon. Butler et al. [2] investigate this modification in a competitive chemostat with periodic dilution rate \( D(t) \) and Monod growth response. They find that if \( \frac{m S^0}{\bar{q} + S^0} > \langle D \rangle^* \langle D(t) \rangle \) is a continuous \( \theta \)-periodic function and \( \langle D \rangle^* = \frac{1}{\theta} \int_0^\theta D(s)ds \) is the mean value, then there are positive \( \theta \)-periodic solutions \( (S(t), x(t)) \) which are exponentially asymptotically stable for the system with the absence of one competitor.

In the aforementioned studies, the chemostat models are described by the system of ordinary differential equations. This is valid only at the macroscopic scale, i.e. the stochastic effects can be neglected or averaged out, on the basis of the law of large numbers. However, the real environment is full of stochasticity, biological models are inevitably affected by environmental noises, which is an important component. Environmental noises will disturb the steady-state of the deterministic system either by directly acting on the population densities or affecting the parameter values. Recently, many stochastic biological models have been developed [11–16]. On the other hand, in many instances, environmental stochasticity also has a critical influence on the nature growth of the biotic populations. Especially for the chemostat, according to [17], every component is inevitably affected by white noise at the microscopic scale. Taking the white noise into account, microorganism systems described by the stochastic differential equations have recently been studied by many researchers [17–22]. In [23], Wang et al. construct a stochastic chemostat with periodic dilution rate and consider the Monod response function. They find the existence conditions for the stochastic nontrivial positive periodic solution and the globally attractive boundary periodic solution, which corresponding to the conclusions in [2]. However, to the best of our knowledge, there is little theoretical results about the nontrivial periodic solution for the stochastic chemostat with periodic dilution rate and general growth responses. So in this paper, we further establish the following stochastic periodic chemostat with general response function:

\[
\begin{align*}
\frac{dS(t)}{dt} &= \left[D(t)(S^0 - S(t)) - \frac{1}{\theta} p(S(t))x(t)\right] \, dt + \sigma_1(t)S(t)dB_1(t), \\
\frac{dx(t)}{dt} &= \left[-D(t) + p(S(t))\right]x(t) \, dt + \sigma_2(t)x(t)dB_2(t),
\end{align*}
\]

where \( B_1(t) \), \( B_2(t) \) are independent standard one-dimensional Brownian motions defined on a complete probability space \( (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P}) \) with a filtration \( \{\mathcal{F}_t\}_{t \geq 0} \) satisfying the usual conditions (i.e. it is right continuous and \( \mathcal{F}_0 \) contains all \( \mathbb{P} \)-null sets), and \( \sigma_i(\sigma_i^2 > 0, i = 1, 2) \) are their intensities, respectively. The parameter functions \( D(t), \sigma_i(t) \) are continuous \( \theta \)-periodic functions and \( D(t) > 0 \). Our aim is to obtain the sufficient criteria for the existence of the stochastic nontrivial positive periodic solution for system (1.4). We establish the stochastic chemostat model (1.4) on the basis of the approach used in [18] to include stochastic effects (readers can also refer to [24, Appendix A] to see the construction of this kind of stochastic model).

This paper deals with the stochastic chemostat (1.4) with general class of growth functions \( p(S) \). It is for the sake of biological reality that we impose the general assumptions for \( p(S) \) above. Again, for the sake of clarity, we make two further classifications of assumptions for the generic nature

\( A_1 \):

(a) \( p \in C^2([0, +\infty), [0, +\infty]) \) and \( \frac{d^2}{dS^2} \leq c, \) for any \( S \in (0, +\infty), \) where \( c \) is a positive constant.

(b) \( p''(S)S^3 \geq m_0, \) for any \( S \in (0, +\infty), \) \( m_0 \) is a constant which requires no assumption on the sign.
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