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On the visibility locations for continuous curves

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ABSTRACT

The problem of determining visibility locations (VLs) on/inside a domain bounded by a planar $C^1$-continuous curve (without vertices), such that entire domain is covered, is discussed in this paper. The curved boundary has been used without being approximated into lines or polygons. Initially, a few observations regarding the VLS for a curved boundary have been made. It is proposed that the set of VLS required to cover the domain be placed in a manner that the VLS and the lines connecting them form a spanning tree. Along with other observations, an algorithm has been provided which gives a near optimal number of VLS. The obtained number of VLS is then compared with a visibility disjoint set, called as witness points, to obtain a measure of the 'nearness' of the number of VLS to the optimum. The experiments on different curved shapes illustrate that the algorithm captures the optimal solution for many shapes and near-optimal for most others.

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1. Introduction

The problem of identifying regions of visibility within a domain (or from outside of it) has been useful in many applications such as mold design for manufacturing, inspection of models, shortest path identification, placing guards to cover an art gallery, sensor location, robot motion planning etc. In the case of mold design, the problem is posed as whether the model is a two-piece, given a set of viewing directions [1]. Alternatively, given a model, the problem is to identify optimal partitioning directions that reduce the number of mold pieces [2,3].

Viewpoint selection that covers the entire object has been used in inspection. Clearly, creating an optimal set of viewpoints (or visibility locations (VLs)) will then reduce the overall cost of inspection (see [4] for a detailed survey on this topic). In the case of shortest path identification [5], the visibility graph has been a very popular construction, which can be computed using tangents [6].

Sensor location also depends on the visibility of a feature, apart from several other factors [7]. Other applications including security, computer graphics (hidden surface removal) etc. also come under the realm of visibility region identification. For further details on the applications, see [8].

Problem Statement, approach and the obtained results

More often than not, in practice, VLS (hereafter, termed as guards, for ease and clarity of explanation) are required to be...
placed not just on the boundary but also interior to it. For example, for a star-shaped curved polygon, as opposed to three guards on the boundary (Fig. 1(a)), a single guard interior to the domain can cover it (as in Fig. 1(b)). In practice, it would be useful if the guards are allowed to be placed interior/on the domain (and not just on its boundary). Hence, given a planar domain bounded by a smooth (i.e. without $C^1$ discontinuities), parametric, non-convex closed simply-connected curve $C(t)$, this paper aims to find the near-optimal number of guards that cover the entire domain. To the best of the knowledge of the authors, this is perhaps the first work aimed in this direction. Vertices are not explicitly defined in the curved boundary considered in this paper, and hence it is different from the curved domains considered in [12,13]. Also, no discretization approach has been employed like in [8], though we use a rule-based approach like the one used in [16]. However, the rules have been arrived upon based on our observations. The approach presented is heuristic-based and greedy in nature. It adds one guard at a time. Moreover, we employ a first order approach to solve this problem, typically employed in hidden Markov models. It can also be noted that there can be many measures to come up with a ‘good’ guard from a candidate set (as can be seen in [16]) and our approach is based on internal tangents as it is related to the visibility in the case of a curved boundary.

We also have proposed an algorithm to compute ‘witness points’, a technique introduced in [16], based on which the optimality of the solution obtained has been conjectured to be no worse than twice the actual optimal number of guards. In practice, based on the experiments conducted, the proposed approach returns the optimal solution for many of the tested curves.

2. Preliminaries

Let the boundary of a domain $D$ be bounded by a parametric closed curve $C(t)$ without $C^1$ discontinuities. Let the exterior of the curve be denoted by $D^c$.

2.1. Definitions

Definition 1. A point on a curve at which the curvature changes sign is called an inflection point.

Definition 2. A point on a curve is concave if its center of curvature and outward normal at that point are in the same direction, otherwise the point is termed convex ($S$ in Fig. 2(a) is concave, whereas $O$ in Fig. 2(b) is convex).

Definition 3. An internal tangent (denoted as IntT) is a line segment completely lying to the interior/on the curve (no point of the line segment lies exterior to the curve) which is a tangent to at least one point on the curve (e.g., the line segment AB in Fig. 2(a) or AO in Fig. 2(b)).

Fig. 1. Boundary guards [8] vs. guard at the interior for a star-shaped domain (guards shown as dots).

Fig. 2. (a) An internatangent $AB$, and (b) its silhouette and occlusion points $S$ and $O$, respectively. (c) Inflection points $I$ and $F$, and their $IPTs \ IO$ and $IF$. (d) The visibility region $\text{Vis}(G)$ of a guard $G$ shown in red color. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Definition 4.** The point at which an internal tangent touches a curve tangentially is called its silhouette point (henceforth denoted by $S$) [8]. $S$ in Fig. 2(a) is the silhouette point of the IntT $AB$.

**Definition 5.** If an internal tangent has another point lying on the curve (apart from its silhouette point), then such a point is called an occlusion point ($O$ in Fig. 2(b)) [8], and is henceforth denoted by $O$.

**Definition 6.** An internal tangent starting at an inflection point is called inflection point tangent (IPT). Its starting point coincides with its silhouette point (Fig. 2(c)).

**Definition 7.** A point $P \in D$ is considered visible to another point $Q \in D$, if for all points $x \in PQ$, $x \cap D^c = \phi$, i.e. no point on the line segment $PQ$ lies completely exterior to the boundary of $D$. Grazing contact is allowed i.e. the line segment can touch the boundary (typically tangentially).

For example, in the Fig. 2(b), $O$ is considered visible to $A$ even though the line segment OA has a grazing contact at $O$.

**Definition 8.** Let $\text{Vis}(G) = \{x \mid x \in D \text{ and } x \text{ is visible to } G\}$ be the set of points forming the visibility region of the point $G$ (the red area in Fig. 2(d) indicates the visibility region of $G$). A set $W$, consisting of points on or inside $C(t)$ are termed as witness points [16] if visibility regions in the set are pairwise disjoint, i.e., $\forall_{\theta, \tau \in W} \text{Vis}(\theta) \cap \text{Vis}(\tau) = \phi$.

2.2. Observations on the visibility of a guard

The motivation for our observations comes from the fact that, unlike a polygonal boundary, a $C^1$ continuous curved boundary does not have explicitly defined vertices. A guard is assumed to be represented as a point which can see in every direction (i.e. has a $360^\circ$ range of visibility). A set of guards is said to cover the domain if every point in the domain is visible to some guard [9]. Also, a guard cannot see through the curved boundary (i.e. the boundary is assumed to be opaque), and can either lie on or interior to it. An
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