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Optimal consumption and portfolio in a jump diffusion market with proportional transaction costs

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Abstract

We consider the problem of optimal consumption and portfolio in a jump diffusion market in the presence of proportional transaction costs for an agent with constant relative risk aversion utility.

We show that the solution in the jump diffusion case has the same form as in the pure diffusion case first solved by Davis and Norman [Mathematics of Operations Research 15 (1990) 676–713]. In particular, we show that (under some assumptions) there is a *no transaction cone D* in the (x, y)-plane such that it is optimal to make no transactions as long as the wealth position remains in D and to sell/buy stocks according to local time on the boundary of D.

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1. Introduction: the model

In this paper we study the problem of optimal consumption and investment policy in a jump diffusion market consisting of a bank account and a stock. A well established model for the stock price is the log-normal diffusion or geometric Brownian motion, which has several computational advantages. The rate of return is independent of the past, stationary (i.e. time-homogenous in law) and follows the normal distribution. In this paper we will drop the latter assumption; returns will not be assumed to be Gaussian. A stochastic process with

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stationary increments independent of the past, and in addition satisfying the mild technical condition of continuity in probability (implying that it has no fixed jump times) is called a *Lévy process*, and is essentially a piecewise Brownian motion with both drift and Poisson jumps with uniform intensity. We briefly note that we pick the unique right continuous (with left limits) version of all processes, which is the natural choice from a stochastic integration point of view; this also applies to the control processes $(\mathcal{L}, \mathcal{M})$ below, ever though that choice may seem less natural from an impulse control point of view.

In view of the above, we shall assume that the bank gives a fixed interest rate r, and the bank deposit then follows the equation

$$\mathrm{d}P_1(t) = rP_1(t)\,\mathrm{d}t, \quad P_1(0) = p_1 \geq 0. \tag{1.1}$$

The price $P_2(t)$ at time t of the stock, is then assumed to be a geometric *Lévy process*, following the stochastic differential equation

$$\mathrm{d}P_2(t) = \alpha P_2(t)\,\mathrm{d}t + \sigma P_2(t)\,\mathrm{d}W(t) + P_2(t^-)\int_{-1}^{\infty} \eta \tilde{N}(\mathrm{d}t, \mathrm{d}\eta), \quad P_2(0) = p_2 \geq 0. \tag{1.2}$$

Here $\alpha \geq r$ and $\sigma > 0$ are constants, and $W(t)$ is a Wiener process (Brownian motion) on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$. The $\tilde{N}$ entity is $\mathcal{F}_t$-centered Poisson random measure,

$$\tilde{N}(t, A) = N(t, A) - \mathsf{E}[N(t, A)] = N(t, A) - tq(A),$$

where $N(t, A)$ is a Poisson random measure measuring the number of jumps with amplitude in (a Borel set) $A \subseteq (-1, \infty)$ up to and including time t. N has a time-homogenous intensity measure $\mathsf{E}[N(t, A)] = tq(A)$; q is then called the *Lévy measure* associated to N. See e.g. Bensoussan and Lions (1984), Jacod and Shiryaev (1987) and Protter (1990) for more information about such stochastic differential equations.

Note that since we only allow jump sizes η which are bigger than -1, the process $P_2(t)$ will remain positive for all $t \geq 0$, a.s., and will not violate limited liability. On the technical side, we shall assume that

$$\int_{-1}^{\infty} 1 \vee \eta^2\,\mathrm{d}q(\eta) < \infty. \tag{1.3}$$

We assume that at any time t the investor can choose a rate $c(t) \geq 0$ of consumption taken from the bank account. We also assume that he can transfer money at any time from one asset to the other with a transaction cost which is proportional to the size of the transaction. Let $X(t)$, $Y(t)$ denote the amount of money invested in asset numbers 1 and 2, respectively. Then the evolution equations for $X(t)$, $Y(t)$ are

$$\begin{aligned}
&\mathrm{d}X(t) = \mathrm{d}X^{c,\mathcal{L},\mathcal{M}}(t) = (rX(t) - c(t))\,\mathrm{d}t - (1+\lambda)\,\mathrm{d}\mathcal{L}(t) + (1-\mu)\,\mathrm{d}\mathcal{M}(t),\\
&X(0^-) = x \in \mathbb{R},\\
&\mathrm{d}Y(t) = \mathrm{d}Y^{\mathcal{L},\mathcal{M}}(t) = Y(t^-)\left(\alpha\,\mathrm{d}t + \sigma\,\mathrm{d}W(t) + \int_{-1}^{\infty} \eta\tilde{N}(\mathrm{d}t, \mathrm{d}\eta)\right) + \mathrm{d}\mathcal{L}(t) - \mathrm{d}\mathcal{M}(t),\\
&Y(0^-) = Y \in \mathbb{R}.
\end{aligned} \tag{1.4}$$