Moment matching machine learning methods for risk management of large variable annuity portfolios

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Variable annuity (VA) with embedded guarantees have rapidly grown in popularity around the world in recent years. Valuation of VAs has been studied extensively in past decades. However, most of the studies focus on a single contract. These methods can be extended neither to valuate nor to manage the risk of a large variable annuity portfolio due to the computational complexity. In this paper, we propose an efficient moment matching machine learning method to solve this problem. This method is proved to be a good candidate for risk management in terms of the speed of and the complexity of computing the annual dollar deltas, VaRs and CVaRs for a large variable annuity portfolio whose contracts are over a period of 25 years. There are two stages for our method. First, we select a small number of contracts and propose a moment matching Monte Carlo method based on the Johnson curve, rather than the well known nested simulations, to compute the annual dollar deltas, VaRs and CVaRs for each selected contract. Then, these computed results are used as a training set for well known machine learning methods, such as regression tree, neural network and so on. Afterwards, the annual dollar deltas, VaRs and CVaRs for the entire portfolio are predicted through the trained machine learning method. Compared to other existing methods Bauer et al. (2008); Gan (2013); Gan and Lin (2015), our method is very efficient and accurate. Finally, our test results support our claims.

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1. Introduction

Variable annuities (VAs) were first introduced in 1970s in United States (Sloane, 1970). They are deferred annuities that are fund-linked during the deferment periods. Beginning in the 1990s, certain guarantees were included in these policies by insurers, such as guaranteed minimal death benefit (GMDB), guaranteed minimal accumulation benefit (GMAB), guaranteed minimal income benefit (GMIB) and guaranteed minimal withdrawal benefit (GMWB), as riders. The GMDB guarantees that the beneficiary of a VA holder receives the greater of the two values: the sub-account value or the total purchase payments, upon the death of the VA holder. The GMAB and GMIB provide accumulation and income protection for a fixed number of

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years being contingent on survival of the VA holder, respectively. The most popular rider among these is the GMWB, which guarantees a specified amount for withdrawal during the life of the contract as long as the amount withdrawn within the policy year and the total amount withdrawn over the term of the policy both stay within a certain limit. For details about these guarantees, the reader can refer to Huang et al. (2014), Ng and Li (2013), Belanger et al. (2009), Bauer et al. (2008).

Due to the new innovation of guarantee schemes, VAs have grown rapidly in popularity around the world in past decades. In 2013 and 2014, the sales of VAs in US are 145 and 140 billion dollars, respectively, according to LIMRA. As a result, almost every insurance company is managing very large VA portfolios. The embedded guarantees associated with VAs are posing a significant financial risk to insurers. Thus, managing the risk of such policies is a crucial issue to these VA providers (Bauer et al., 2012).

The academic literatures on valuing and hedging guarantees in VA contracts are extensive. Armstrong (2001) studied the guarantee reset features in segregated funds using a discrete time Markov chain model. Boyle et al. (2001) proposed a Monte Carlo method to valuate the reset options embedded in some segregated funds. Milevsky and Promislov (2001) used risk-neutral option pricing theory to price GMDB in a VA contract. Coleman et al. (2006) used local risk minimization to study the discrete hedging of the guarantees embedded in a VA contract with both equity risk and interest rate risk. They concluded that hedging with standard options is better than hedging with the underlying assets. Boyle and Tian (2008) analyzed the design of general equity-indexed annuity from the investor's perspective and proposed a generalization of the conventional design. Lin et al. (2009) used the Esscher transform to determine an equivalent martingale measure for the fair valuation of a VA contract under a regime-switching model in the incomplete market setting. Xu and Wang (2009) proposed a model based on a two-dimensional partial differential equation to price the GMWB rider. Gao and Ulm (2012) studied the valuation of the GMBD rider using a utility-based approach. Yang and Dai (2013) proposed a tree model to price the GMWB rider embedded in deferred life annuity contracts.

Unfortunately, existing valuation/hedging methods used in the risk management of an individual VA contract cannot be feasibly extended to a large VA portfolio. There are two reasons for that. One reason is that the complexity of the payoff function does not lead to closed-form formulas to evaluate the liability of guarantees. The other reason is that the valuation and calculation sensitivity is computationally challenging when the contract number is large. In fact, determining how to hedge the risk of a large VA portfolio and the corresponding required capital poses a significant computational challenge to insurance companies (Bauer et al., 2008).

In practice, insurance companies typically follow a market-to-model approach and rely heavily on simulations. Nested simulations (NS) are used to determine the probability distribution of loss from mismatching in order to calculate the required capital (Reynolds and Man, 2008). There are two levels for the NS procedure. The first level, called the outer loop, projects the VA liabilities onto real world scenarios. The second level, the inner loop, projects the liability of a VA contract onto a large number of simulated risk-neutral paths at each node of an outer loop. A significant concern with NS is its heavy computational cost. For example, if we use 1000 paths for 30 years at each annual node of a 30-year VA contract over 1000 scenarios, then we end up with a computational problem with $1000 \times 30 \times 1000 = 30$ million simulations. If the portfolio consists of 100,000 contracts, it means $3 \times 10^{13}$ simulations. This represents a lot of computations!

Effective ways to reduce the computational time are to reduce either the number of outer loop scenarios or the number of inner loop paths. However, because the Monte Carlo method heavily depends on the number of simulations, too few outer loop scenarios or inner loop paths would result in inaccuracy in computation. One approach to reduce the heavy computing demands in nested simulations is to replicate portfolios. Daul and Vidal (2009) studied the quadratic programming method to replicate cash flows of life insurance liabilities in general. Dembo and Rosen (1999) presented a portfolio replication framework that minimizes the sum of absolute differences instead of the sum of squared differences. However, both methods are still quite expensive when being used to replicate a large portfolio. Another approach is to regress the liability value on some key economic factors (Cathcart and Morrison, 2009). Then, the least square Monte Carlo method is employed to approximate the future liability at each time step. This approach can significantly reduce the number of inner loop paths, but effectively determining the basis functions in the least square Monte Carlo (Longstaff and Schwartz, 2001) is not trivial in real applications.

Gan (2013) proposed a method based on the k-prototypes data clustering combined with the ordinary kriging method to price guarantees for a large portfolio of VAs. The speedup of this method is significant, but it is only developed for one level of simulations, rather than nested simulations. In 2015, Gan and Lin (2015) extended the data clustering method to compute annual dollar deltas of a large VA portfolio, which does require nested simulations. In their method, k representative contracts are selected whose dollar deltas are computed by nested simulations. Then, annual dollar deltas for the reminder of VA contracts in the portfolio are determined by the universal kriging for function data (UKFD) method. In Gan and Lin (2015), only 5 outer loop scenarios are considered in the nested simulations. Significant computational time is required when 1000 outer loop scenarios are considered to compute the annual dollar deltas of a large VA portfolio because computing annual dollar deltas for k representative contracts is still expensive through the nested simulations (even though k is relatively small compared to the size of the portfolio).

In this paper, we propose an efficient moment matching machine learning (MMML) method for one-factor market scenario models to evaluate deltas, VaRs and CVaRs for a large VA portfolio. The real world scenarios are more complicated

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