The fundamental law of active management: Redux

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ABSTRACT

We develop a fundamental law of active management based on cross-section factor models for residual returns where the latter have unconditional mean zero and the factor exposures have zero mean and unit variance. Under our model framework the factor returns are cross-sectional information coefficients ICt that vary randomly over time with constant mean and variance. The fundamental law holds for portfolio managers who use conditional expectation of the residual returns and the associated conditional covariance matrix as inputs to active quadratic utility portfolio optimization. The fundamental law formula shows that the optimal portfolio’s information ratio (IR) is positively related to the mean of ICt and the number of assets N in the portfolio selection universe, inversely related to the volatility of ICt, and is an absolute upper bound on IR as N tends to infinity. Support for our choice of factor model and our fundamental law is provided by an empirical study showing that significantly higher IR values are obtained using our choice of factor model as compared with IR values using an industry standard factor model.

1. Introduction

In his seminal work Grinold (1989) introduced the concept of information ratio and stated without proof a “fundamental law of active management” in the form of a simple formula for the information ratio. We recall that the information ratio (IR) of an actively managed portfolio is defined as the portfolio’s mean return in excess of a benchmark portfolio return divided by the tracking error (standard deviation of the excess return). Since its initial introduction the IR has played a fundamental and ubiquitous role in gauging the performance of an active portfolio manager who manages a portfolio relative to a benchmark. Goodwin (1998) provides a useful reference on IR basics. A more well-developed version of the ideas in Grinold (1989) play a central role in the Grinold and Kahn (2000) book Active Portfolio Management (G & K) that became the “bible” on the topic for many active portfolio managers for many years.

The beginning of Chapter 6 in G & K introduces the fundamental law of active management (FLAM), and states that the information ratio is approximated by the formula IR = IC ⋅ √BR where IC is the “information coefficient”, and BR is “breadth” defined to be the number of independent forecasts of exceptional returns made each year. Here “exceptional” means the returns obtained after removing beta-adjusted benchmark returns. The term information coefficient is simply a colorful way of referring to a correlation coefficient, namely the correlation between asset returns and the predictor signal used by the manager to forecast the returns. Here asset returns refers to “residual” returns as defined subsequently, and also referred to as “exceptional” returns in G & K. Once one realizes that an IC is a correlation coefficient between returns and a predictor of returns it is not surprising that Grinold identified the IC as a measure of skill. More skilled active managers find better signals that have larger correlation with the returns to be predicted and smaller prediction errors, leading to more profitable portfolios.

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The Chapter 6 Appendix of G & K combines the quadratic utility method of finding optimal active portfolio weights with best linear predictor theory to derive an optimal information ratio. But then the fundamental law as stated was taken without justification to be for the very special case that “all signals have equal value”, i.e., there is a common correlation for all predictors. Taken as a totality there are several problems with the above results. First of all, being based on generic best linear predictor theory the law is not based on any econometric data-generating model as a foundation. Second, although the derivation makes use of the conditional mean predictors it does not make appropriate use of the conditional forecast error covariance structure. Finally, the law as stated above does not provide any guidance on how one should calculate the number of independent forecasts. As a consequence, practitioners have sometimes taken breadth to be the number of assets in the portfolio selection universe, and this leads to IR values that are far too optimistic relative to what active managers can achieve in practice.

Subsequent to G & K a number of authors have tried to improve the formulation of the fundamental law. Buckle (2004) again used a best linear predictor framework along with some of the G & K assumptions to derive a more general form of the law involving predictor correlations that reduces to the G & K simple form when the predictors are independent. Grinold (2007) also made some proposals on measuring breadth. Clarke et al. (2002) extended the law to take into account portfolio optimization constraints, but under an unrealistic assumption that residual returns are uncorrelated. Clarke et al. (2006) extended the law to the case of correlated assets with constraints. The results show not unexpectedly that the impact of constraints is to reduce the information ratio. A completely new and important fundamental law ingredient was introduced by Qian and Hua (2004). They derived a fundamental law that reflects the randomness of the IC over time by virtue of the appearance of the IC mean and standard deviation in their fundamental law, and referred to this randomness as “strategy risk”. Subsequently Ye (2008) derived a version of the law that took into account randomness of the IC.

In spite of the considerable literature on the fundamental law and its extensions, there exist basic limitations of the existing derivations of the law. The first and most basic is that none of the derivations use an econometric data-generating model as a foundation. Second, most of the approaches lack a well-defined model for IC randomness. Last but not least, the lack of adequate treatment of conditional versus unconditional variance and covariance in the derivations results in overly optimistic information ratios for the resulting optimized portfolios. Our work removes these limitations by: (1) using an econometric factor model motivated abstractly by the arbitrage pricing theory and concretely by the well-established industry use of cross-section fundamental factor models in asset, but with a significant difference from the latter, (2) endowing the econometric model with a key randomness component that results in IC randomness, and (3) proper handling of conditional and unconditional predictions and their covariance. As a result we obtain a new general form of fundamental law of active management based on a solid foundation that contains prior versions as special cases.

In Section 2.1 we discuss residual returns from a benchmark single index model and a CAPM motivation for a key property of residual returns. Section 2.2 defines residual returns conditional mean forecast and related conditional forecast error covariance matrix and Section 2.3 describes the quadratic utility active portfolio optimization method used in developing the fundamental law. Section 2.4 defines and discusses the multi-factor model assumptions we use, and proves two results used in subsequent development of the fundamental law. Section 3 focuses on single-factor models, defines factor returns and identifies them as time series of random information coefficients and derives the single factor fundamental law. A key result is that there is an absolute upper bound to the information ratio no matter how many assets the portfolio includes, and it is shown that the G & K formula is an unrealistic limiting case of our single factor fundamental law. Section 4 extends the single factor model results to the more general case of a multi-factor model, and briefly discusses the negative impact of using a factor model that is misspecified by virtue of leaving out some needed factors. Section 5 provides discussion and results of a detailed empirical study in support of our new fundamental law for the case of a single factor model. Section 6 provides a summary and concluding comments on future research needed.

2. Assumptions and discussion

The four subsections to follow discuss four important sets of framework assumptions we use in deriving the fundamental law of active management. The first has to do with the use of residual returns relative to a benchmark as the input to portfolio optimization. The second is concerned with the use of conditional mean forecasts and the associated conditional covariance matrix. The third concerns our use of portfolio optimization based on quadratic utility optimization. The fourth subsection provides our assumptions concerning the multi-factor model used along with related discussion of the assumptions.

2.1. Benchmark residual returns

The following single index model relative to the benchmark is a standard tool in active portfolios management:

\[ r_{it}^{Total} = \beta_i r_{t,B} + r_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T, \tag{1} \]

where

\[ r_{it}^{Total} = \text{total return of security } i \text{ at time } t \text{ in excess of the risk-free rate} \]
\[ \beta_i = \text{beta of security } i \text{ with respect to the benchmark} \]
\[ r_{t,B} = \text{benchmark return at time } t \text{ in excess of the risk-free rate} \]
\[ r_{it} = \text{residual return of security } i \text{ at time } t \]

and the residual returns are uncorrelated with the benchmark returns. The benchmark is assumed to be a broad market proxy index, e.g., the Russell 3000 or Wilshire 5000 for the U.S. market and the MSCI All-World Index for the global market.
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