Maximization of Returns under an Average Value-at-Risk Constraint in Fuzzy Asset Management

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Abstract

A portfolio allocation model is discussed in asset management with fuzziness. By perception-based extension for fuzzy random variables, the estimation methods of asset risks are introduced. Introducing an average value-at-risk for fuzzy random variables, this paper formulates a portfolio allocation model with average value-at-risks. This paper discusses maximization of the expected return under an average value-at-risk constraint with fuzzy random variables. A numerical example is given to demonstrate the results.

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1. Introduction

In financial asset management, portfolio allocation is a technique not only for minimization of management risks but also for maximization of expected returns. In classical Markowitz’s mean-variance portfolio model, the variance is demonstrated as a risk in asset management (Markowitz 1, Pliska 2, Steinbach 3). Recently drastic declines of asset prices are studied, and average value-at-risks (AVaR) is used to estimate the risk when asset prices decline rapidly with worst scenarios (Chun et al. 4, Rachev et al. 5, Wang et al. 6). AVaR is one of coherent risk measures in financial management and it is defined by percentiles at a specified probability.

Fuzzy random variables, which were introduced by Kwakernaak 7, are applied to decision making under uncertainty with fuzziness such as linguistic data in engineering, economics et al. To represent uncertainty, we use fuzzy random variables which have two kinds of uncertainties, i.e. randomness and fuzziness. In this paper, randomness is used to represent the uncertainty regarding the belief degree of frequency, and fuzziness is applied to linguistic imprecision.

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of data because of a lack of information about the current stock market. When the financial crisis in October 2008 and January 2016, we have experienced the serious distrust regarding the stock market because panic with some risks occurs from the imprecision of information. In this paper, using fuzzy random variables we deal with optimization of portfolio allocation in an environment with both randomness and fuzziness.

We extend the AVaR for real random variables to one regarding fuzzy random variables from the viewpoint of perception-based approach in Yoshida, and we discuss perception-based criteria for the uncertainty. Yoshida introduced the mean, the variance and the covariances of fuzzy random variables, using evaluation weights and \( \lambda \)-mean functions. This paper estimates fuzzy numbers and fuzzy random variables by probabilistic expectation and these criteria, which are characterized by possibility and necessity criteria for subjective estimation and pessimistic–optimistic indexes for subjective decision. These parameters are decided by the investor with his certainty about information in the stock market. In this paper we discuss maximization of expected returns under an AVaR constraint with fuzzy random variables based on the results in Yoshida.

In Section 2, we introduce AVaR estimation for fuzzy random variables by perception-based extension. In Section 3, we give estimation tools with evaluation weights and \( \lambda \)-mean functions in order to evaluate the randomness and fuzziness of fuzzy random variables. In Section 4, we investigate the upper bound of AVaRs regarding the rate of fuzziness of fuzzy random variables. In Section 5, we discuss maximization of the expected returns with fuzziness to give a reasonable AVaR constraint. In Section 6, we discuss maximization of expected returns under an AVaR constraint with fuzzy random variables. In Section 7, we demonstrates a numerical example to explain the results in previous sections.

2. Fuzzy random variables and average value-at-risks

Let \( \mathbb{R} \) be the set of all real numbers. A fuzzy number is represented by its membership function \( \tilde{a} : \mathbb{R} \to [0, 1] \) which is normal, upper-semicontinuous, fuzzy convex and has a compact support (Zadeh). Let \( \mathcal{R} \) be the set of all fuzzy numbers. For a fuzzy number \( \tilde{a} \in \mathcal{R} \), its \( \alpha \)-cuts are given by \( \tilde{a}_\alpha = \{ x \in \mathbb{R} | \tilde{a}(x) \geq \alpha \} \) (\( \alpha \in (0, 1) \)) and \( \tilde{a}_0 = \text{cl}(\{ x \in \mathbb{R} | \tilde{a}(x) > 0 \}) \), where \( \text{cl} \) denotes the closure of an interval. The \( \alpha \)-cut is also written by closed intervals \( \tilde{a}_\alpha = [\tilde{a}_\alpha^-, \tilde{a}_\alpha^+] \) (\( \alpha \in [0, 1] \)). Hence we introduce a partial order \( \succeq \), so called the fuzzy max order, on fuzzy numbers \( \mathcal{R} \). Let \( \tilde{a}, \tilde{b} \in \mathcal{R} \) be fuzzy numbers. \( \tilde{a} \succeq \tilde{b} \) means that \( \tilde{a}_\alpha \succeq \tilde{b}_\alpha \) for all \( \alpha \in [0, 1] \). An addition and a scalar multiplication for fuzzy numbers are defined by as follows: For \( \tilde{a}, \tilde{b} \in \mathcal{R} \) and \( \lambda \in \mathbb{R} \) satisfying \( \lambda \geq 0 \), the addition \( \tilde{a} + \tilde{b} \) of \( \tilde{a} \) and \( \tilde{b} \) and the nonnegative scalar multiplication \( \lambda \tilde{a} \) of \( \lambda \) and \( \tilde{a} \) are fuzzy numbers given by their \( \alpha \)-cuts \( (\tilde{a} + \tilde{b})_\alpha = [\tilde{a}_\alpha^- + \tilde{b}_\alpha^-, \tilde{a}_\alpha^+ + \tilde{b}_\alpha^+] \) and \( (\lambda \tilde{a})_\alpha = [\lambda \tilde{a}_\alpha^- , \lambda \tilde{a}_\alpha^+] \), where \( \tilde{a}_\alpha^+ \), \( \tilde{a}_\alpha^- \), \( \tilde{b}_\alpha^- \) and \( \tilde{b}_\alpha^+ \) (\( \alpha \in [0, 1] \)). Let \( P \) be a non-atomic probability measure on a sample space \( \Omega \). Let \( X \) be the family of all integrable real random variables on \( \Omega \) such that there exists a continuous cumulative distribution function \( x \mapsto F_X(x) = P(X < x) \) for which there exists a non-empty open interval \( I \) such that \( F_X(I) : I \to (0, 1) \) is a strictly increasing and onto. Then there exists a strictly increasing and continuous inverse function \( F_X^{-1} : (0, 1) \to I \). We note that \( F_X(I) : I \to (0, 1) \) and \( F_X^{-1} : (0, 1) \to I \) are one-to-one and onto, and we have \( \lim_{x \to \inf \cdot} F_X(x) = 0 \) and \( \lim_{x \to \sup \cdot} F_X(x) = 1 \). Hence, the value-at-risk (VaR) at a probability \( p \in (0, 1) \) is given by the percentile of the distribution function \( F_X \) (Yoshida):

\[
\text{VaR}_p(X) = \sup \{ x \in I | F_X(x) \leq p \} = F_X^{-1}(p).
\]

Then we have \( F_X(\text{VaR}_p(X)) = p \). Further, the average value-at-risks (AVaR) at a probability \( p \in (0, 1) \) is given by

\[
\text{AVaR}_p(X) = \frac{1}{p} \int_0^p \text{VaR}_q(X) dq.
\]

It is known that AVaR has the following properties (Yoshida), which imply AVaR is a coherent risk measure (Artzner et al.).

**Lemma 1.** Let \( X, Y \in \mathcal{X} \) be real random variables and and let a probability \( p \in (0, 1) \). Then the average value-at-risk AVaR\(_p\) has the following properties:

(i) If \( X \leq Y \), then \( \text{AVaR}_p(X) \leq \text{AVaR}_p(Y) \). (monotonicity)
(ii) \( \text{AVaR}_p(\zeta X) = \zeta \text{AVaR}_p(X) \) for \( \zeta > 0 \). (positive homogeneity)
(iii) \( \text{AVaR}_p(X + \theta) = \text{AVaR}_p(X) + \theta \) for \( \theta \in \mathbb{R} \). (translation invariance)
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