Generalized interval-valued OWA operators with interval weights derived from interval-valued overlap functions

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\textbf{A B S T R A C T}

In this work, we extend to the interval-valued setting the notion of overlap functions, presenting a method which makes use of interval-valued overlap functions for constructing OWA operators with interval-valued weights. Some properties of interval-valued overlap functions and the derived interval-valued OWA operators are analyzed. We specially focus on the homogeneity and migrativity properties.

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\section{1. Introduction}

Interval-valued fuzzy sets [77] have been successfully applied in many different problems. Just to mention some of the most recent ones, interval-valued fuzzy sets have been used in decision making (see, e.g., the works by Khalil and Hassan [44] and Cheng et al. [22]), image processing (see, e.g., the works by Barrenechea et al. [3], Pagola et al. [53] and Melin et al. [47]), prediction (e.g., the work by Rodrigues et al. on the prediction of pests in agriculture) or classification (see, e.g., the works by Sanz et al. [67,68]). They have also been the origin of rich theoretical studies, as, for instance, the works by Bedregal et al. [4,9], Dimuro et al. [27,32,33], Reiser et al. [59] and the recent works by Zywica et al. [79] and Takác [70].

From the point of view of applications, interval-valued fuzzy sets are a suitable tool to represent uncertain or incomplete information. In particular, the length of the interval-valued membership degree of a given element can be understood as a measure of the lack of certainty of the expert for providing an exact membership value to that element [32]. Interval degrees are also be used to summarize the opinions of several experts. In this case, the left and right interval endpoints can be, for instance, the least and the greatest membership degrees provided by a group of experts. This makes interval-valued fuzzy sets very useful for multieexpert decision making problems, when the experts are asked to express numerically their preferences on several alternatives, as discussed by Bustince et al. [19] (see also the discussions about that in [4,11,17,18]).
Besides, another relevant tool for many different application is that of OWA operators, introduced by Yager [74] and largely used in the literature (see, e.g., [42,43,49]). Its usefulness has led to the consideration of different possible extensions for Atanassov intuitionistic fuzzy sets ([45,50,73,76]) and for interval-valued fuzzy sets ([19,23,72,78]).

In the latter case, however, one of the key problems is how to build and normalize interval-valued weights. In the literature, interval-valued weights are used in several contexts, in order to face the problem of real-world applications in which there are a lot of uncertainty involved and lack of consensus among the modeling experts. Pavlacka [56] presented a review of the existing methods for normalization of interval weights. For example, in the context of multi-criterion decision making, Wang and Li [71] used a hierarchical structure to aggregate local interval weights into global interval weights, by means of a pair of linear programming models to maximize the lower and upper bounds of the aggregated interval value.

However, in the definitions of interval-valued OWA operators found in the literature, the weighting vector is composed, in general, by real numbers. Due to this limitation of the actual models of interval-valued OWA operators, in this paper, we propose the use of interval weights. In order to define these weights we propose the extension of the so-called overlap functions [10,13,16,28,30,31,58] to the interval-valued setting. In this way, the normalization method proposed here makes use of the properties of aggregation functions, and, thus, it is defined in flexible terms.

Then, the objectives of this paper are:

• To introduce the concept of interval-valued overlap functions, and to analyze some of its most relevant properties, such as migrativity and homogeneity;
• To define the normalization of an interval-valued weighting vector by means of a general aggregation function, and to determine which conditions normalized weighting vectors should fulfill;
• To develop a construction method of interval-valued OWA operators based on interval-valued overlap functions, considering interval-valued weights;
• To study the properties of such OWA operators, specially considering the migrativity and homogeneity of interval-valued overlap functions.

This work is organized as follows. In Section 2, we recall some basic concepts that are used along the paper. Next, we define the basic order relations between intervals and, in Section 4, we introduce the concept of interval-valued overlap function, studying some of its most important properties. In Section 5, we present the concept of normalized weighting vector, analyzing the definition of interval-valued OWA operators with interval weights. We also study the conditions which the functions used for this definition must fulfill to recover idempotency and other properties. We finish with conclusions and references.

2. Preliminary concepts

We start, in this section, recalling some well-known important concepts that are used in our subsequent developments. Consider a function \( f : [0,1]^n \rightarrow [0,1] \). Given \( i \in \{1,\ldots,n\} \), we say that the component \( i \) is necessary if there does not exist a function

\[
g : [0,1] \times \ldots \times [0,1] \times [0,1] \times \ldots \times [0,1] \rightarrow [0,1]
\]

such that

\[
f (x_1, \ldots, x_n) = g(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n),
\]

for any \( (x_1, \ldots, x_n) \in [0,1]^n \).

2.1. Aggregation functions

A crucial concept for the present paper is that of aggregation function (see [20]).

Definition 2.1. A \( n \)-ary aggregation function is a mapping \( M : [0,1]^n \rightarrow [0,1] \) such that

(M1) \( M(0, \ldots, 0) = 0 \) and \( M(1, \ldots, 1) = 1 \);

(M2) \( M \) is increasing in each argument: for every \( i = 1, \ldots, n \), if \( x_i \leq y_i \) then \( M(x_1, \ldots, x_n) \leq M(y_1, \ldots, y_n) \).

Several other properties can be required for aggregation functions. In particular, in this work we are interested in the following two ones.

(M3) If \( M(x_1, \ldots, x_n) = 0 \) then there is \( i = 1, \ldots, n \) such that \( x_i = 0 \);

(M4) If \( M(x_1, \ldots, x_n) = 1 \) then there is \( i = 1, \ldots, n \) such that \( x_i = 1 \).

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