Control of Brokerage Margins

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Abstract: In this paper we analyze financial risk from the point of view of a brokerage company, who exposes itself to risk by lending assets or money to its clients for allowing short-selling or leveraged operations (firm-wise risk). We develop analytical models for control of firm-wise risk, by defining both specific margin factors for single assets and a global margin factor that takes into account the overall riskiness of a complex portfolio. In the first part of this work we derive a model to evaluate leverage factors, by linking them with the probability for the client’s portfolio value to go below a certain safety threshold, using Value-at-Risk and Expected Shortfall approaches. Further, we present optimization models based on these two approaches in order to determine the optimal leverage factors. In the second part, we present a model for margin control based on the concept of marginal availability. A global margin factor considering the overall riskiness of a complex portfolio is derived, and we show the effectiveness of the approach also when dealing with portfolios containing options.

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1. INTRODUCTION

Buying on margin consists in borrowing money from a broker for purchasing securities. Similarly, short-selling a security consists in borrowing that security from the broker in order to sell it immediately on the market. In both cases, the investor should put on the table a margin, that is a fraction of the investment value, and the debt with the broker for the residual amount should be cleared at the end of the trading day. Such loans from the brokerage company allow the investor to apply financial leverage to her investments, thus magnifying both gains and losses.

While a number of methodologies exist to evaluate the margin requirement for a given position, they tend to be categorized into one of two approaches: rules based or risk based, see e.g., Barry (2012). Rule-based methods generally assume uniform margin rates across similar products, offer no inter-product offsets, and consider derivative instruments in a manner similar to that of their underlying security. In this sense, they offer ease of computation but oftentimes make assumptions which, while simple to execute, may overstate or understate the risk of an instrument relative to its historic performance. In contrast, risk-based methodologies seek to apply a margin coverage that is related to the product’s past performance. They recognize some inter-product offsets, and seek to model the nonlinear risk of derivative products using mathematical pricing models, see Eurex (2011). These methodologies involve computations which may not be easily replicable by the client. Moreover, to the extent that their inputs rely upon observed market behavior, they may result in requirements that are subject to rapid and sizable fluctuation. Examples of risk-based methodologies include TIMS (Theoretical Intermarket Margin System) and SPAN (Standardized Portfolio Analysis of Risk Margin), see An.Vv. (1996) and CME Group (2010) for more details about these models.

Most online brokerage companies or banks do provide their clients with the possibility of buying on margin. However, the margins offered to the general public are usually fixed and conservative. Indeed, the brokerage company typically has a list of assets on which margin is allowed, and their corresponding margins, for example, say, a 20% margin is required on ENI, a 30% margin is required on FCA, a 10% margin is required on Generali, etc. Such values are held fixed for long periods of time, and changed occasionally, as decided by the board of directors. While, on the one hand, such an approach may be safe for the brokerage company, on the other hand it may render the company less competitive on the market, since active traders may go away towards other brokers that allow for less conservative margins. The need thus arises to provide the client with dynamic margins, tailored ad hoc on the client’s overall position, and computable “on the fly,” as the request for a transaction is received by the broker.

In this paper we introduce two novel risk-based models for computing margin factors in real time. While both TIMS and SPAN are mainly based on the simulation of hypothetical market scenarios and take the largest loss as the margin requirement, the models that we present link margin factors with the overall portfolio riskiness by means of quantitative risk measures such as Value at Risk (VaR), Rachev et al. (2008), and Expected Shortfall (ES), Meucci (2005). The main advantage of this approach is that it allows for closed-form expressions for margin factors, which can be used by the firm also to set up optimal portfolio strategies and to achieve more competitive levels of margin rates while keeping the risk under control. Moreover, in contrast to rules-based methods, correlations and diversification effects are automatically taken into account.

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when evaluating the overall portfolio risk through VaR (for certain probability distributions) or ES.

In this paper we shall discuss almost indistinctly about margin or leverage factors, since they refer essentially to the same concept. In particular, the leverage factor is the inverse of the margin, and vice versa. The difference in terminology is essentially related to different client/broker points of view, the client being typically more interested in the available leverage, with the broker focusing more on the margin to be requested.

2. BASIC CONCEPTS AND NOTATION

In this section we introduce some concepts and notations that will be used throughout this paper.

Let \( P_t - \Delta t \) and \( P_t \) be the prices of an asset at the beginning and at the end of a time period \( \Delta t \) respectively. The simple return of the asset over the period \( \Delta t \) is given by: \( R_{S \Delta t} := P_t / P_{t-\Delta t} \). The rate of return (or just return), instead, is a profit on an investment over a period of time, expressed as a proportion of the original investment: \( R_{\Delta t} := (P_t - P_{t-\Delta t}) / P_{t-\Delta t} = P_t / (P_{t-\Delta t}) - 1 \). Notice that we have \( R_{S \Delta t} = 1 + R_{\Delta t} \). In the following we will consider daily returns, in the sense that \( \Delta t = 1 \), and thus we will drop the \( \Delta t \) notation.

A portfolio of assets is defined by a vector \( x \in \mathbb{R}^n \) whose \( i \)-th entry \( x_i \) describes the (signed) amount of an investor’s wealth invested in the \( i \)-th asset.

Let \( L \) be the random variable describing the loss on a given portfolio over a given time interval, then the \( \alpha \)-level Value at Risk is defined as \( \text{VaR}_\alpha(L) := \inf\{l \in \mathbb{R} : \mathbb{P}(L > l) \leq \alpha \} = \inf\{l \in \mathbb{R} : F_L(l) \geq 1 - \alpha \} \). Here, \( F_L(l) \) is the cumulative distribution function of the variable \( L \) evaluated in \( l \). The VaR can be equivalently expressed in terms of the random payoff \( X = -L \) as follows:

\[
\text{VaR}_\alpha(X) = -\sup_x \{x \in \mathbb{R} : \mathbb{P}(X < x) \leq \alpha \}. \tag{1}
\]

The evaluation of the VaR of an asset or portfolio is related to the VaR of its return. Indeed, if \( X \) is the end-of-day (eod) value of an asset (or of a portfolio), \( X_0 \) its present value (known) and \( R \) its return, then \( X = (1 + R)X_0 \), and \( \text{VaR}_\alpha(X) = X_0(\text{VaR}_\alpha(R) - 1) \). The \( \alpha \)-level Expected Shortfall of a random payoff \( X \) is defined as

\[
\text{ES}_\alpha(X) := -\frac{1}{\alpha} \int_0^\alpha \text{VaR}_\alpha(X) \, dp = -\frac{1}{\alpha} \int_0^\alpha F_X^{-1}(\alpha) \, d\alpha. \tag{2}
\]

Notice that this can be equivalently written in terms of a conditional expectation: \( \text{ES}_\alpha(X) = -\mathbb{E}[X \mid X < \text{VaR}_\alpha(X)] \), where the latter equality holds true if and only if the cumulative density function of \( X \) is continuous at \( x = \text{VaR}_\alpha(X) \). Notice that, by definition, it holds \( \text{ES}_\alpha(X) \geq \text{VaR}_\alpha(X) \) \( \forall X, \forall \alpha \in [0, 1] \).

3. LEVERAGE CONTROL – SINGLE ASSET CASE

In this section we shall develop two models for leverage control for a single asset, based on Value At Risk and Expected Shortfall, respectively. This is useful in order to define single margin factors that the brokerage company may impose on its clients.

Suppose that a client wants to invest a fraction \( w \) of his capital \( C \) on a some risky asset, whose daily return is given by the random variable \( R \). The remaining fraction of the money, namely \( 1 - w \), is kept by the client as a reserve. The firm allows the client to borrow money up to \( l_R \) times the value of the original investment on the asset, so that with an upfront payment of \( wC \) the client buys the stock for a value \( l_R w C \). The money borrowed by the client is then \( C l_R w - C w (l_R - 1) \). The client portfolio at the end of the day (eod) has the (random) value \( C_+ = C (l_R w (1 + R) + 1 - w) \). At the end of the day, however, the client must pay back his debt, which means that the net amount remaining will be: \( C_+ = C_+ - C w (l_R - 1) = C (l_R w R + 1) \). We shall consider the ratio between \( C_+ \) and the invested amount \( l_R w C \). Thus we define:

\[
\Delta = \frac{l_R w R + 1}{l_R w} \tag{3}
\]

From the point of view of the brokerage company, we want this \( \Delta \) to be greater than a certain safety threshold \( h \). Leverage control thus amounts to finding a suitable range for the leverage factor \( l_R \) such that the condition \( \Delta > h \) holds with some suitably high probability.

3.1 VaR-based approach

A natural way of choosing the value of \( l_R \) would be to ask that the probability for the client’s \( \Delta \) to go under the threshold \( h \) is small enough. Namely, if we fix a confidence level \( \alpha \in (0, 1) \), we want that

\[
l_R \in \{l_R \geq 1 : \mathbb{P}(\Delta \leq h) \leq \alpha \}, \tag{4}
\]

which, using (3), becomes

\[
\mathbb{P}(\Delta \leq h) = \mathbb{P}(R \leq h - \frac{1}{l_R w}).
\]

In order to make the condition in (4) explicit, we need a probabilistic model for the asset return \( R \). A common approach is to assume that \( R \) is normally distributed, with some given mean \( \mu_R \) and variance \( \sigma_R^2 \), i.e., \( R \sim N(\mu_R, \sigma_R^2) \). In this case, if we define \( Z \sim N(0, 1) \) as the standard normal variable and \( q_\alpha \) as the \( \alpha \)-level quantile, then by standardizing the variable \( R \), we have

\[
\mathbb{P}
\left(Z \leq \frac{l_R w h - 1 - l_R w \mu_R}{l_R w \sigma_R}
\right)
\leq \alpha \Rightarrow \frac{l_R w h - 1 - l_R w \mu_R}{l_R w \sigma_R} \leq q_\alpha \text{ which implies } l_R \leq \frac{w(h - \mu_R - q_\alpha \sigma_R)}{1 - q_\alpha \sigma_R}.
\]

We have that \( \mu_R + q_\alpha \sigma_R = \mu_R - q_{1-\alpha} \sigma_R = -\text{VaR}_\alpha(R) \), and plugging this in the above formula we finally obtain

\[
l_R \leq \frac{1}{w(h + \text{VaR}_\alpha(R))} \tag{5}
\]

This formula links in a very simple way the maximum leverage factor with the asset riskiness, measured in this case by the VaR, and the amount of money invested on it. We can observe that, since it must be \( l_R \geq 1 \), we can say that if the \( l_R \) value resulting from (5) is \( < 1 \), then it means that the asset is too risky, and the firm shall not offer a leverage on it. Obviously, a critical value for the threshold is \( h = 0 \). If \( \Delta \) falls below this value, it means that the client does not have enough money to pay back his debt. Since the upper bound in (5) decreases as \( w \) increases, a safe approach is to take \( w = 1 \) for determining conservative margins.

3.2 Expected shortfall approach

The VaR-based method that we discussed in the previous section has the usual drawback related to VaR, that is,
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