Pricing of basket options in subdiffusive fractional Black–Scholes model

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1. Introduction

During the last few decades the scientists have had a keen interest to the problem of financial markets' modelling and derivation of prices of different financial derivatives. The breakthrough happened when in 1973 the papers of Black and Scholes [1] and Merton [14] were published. These papers had a great impact to the financial market and arouse huge academic interest, setting up the key principles of arbitrage option pricing. However, the original model has its disadvantages as it sets a number of restrictions on the real state of life [19]. Many of assumptions were relaxed in the ongoing researches. In this work the subdiffusive phenomena is of specific interest.

The fact is that in some financial markets the number of participants and, therefore, performed transactions, is so low that the price of the asset may remain constant during the period of time. This phenomenon is the most inherent for the emerging markets and it breaks the assumption on liquidity set in the original Black–Scholes (B–S) model. The idea to model the price of such assets using subdiffusive Geometric Brownian Motion (GBM) comes from physics: the so-called stagnation periods in the market are associated with the trapping events of the subdiffusive test particle [4], which is manifested by the fractional derivatives in Fokker–Planck equations [15,16].

The generalized to the subdiffusive regime B–S model for one dimensional case was already introduced in details in the literature [9]. However, such model cannot define the price of such important financial derivatives as, for instance, European basket options. For these reasons the multidimensional B–S model was applied in the standard approach. Here we generalize the multidimensional B–S model by adding the periods of stagnation, which are characteristic for subdiffusion and can be modelled by equations involving fractional derivatives. We underline the extension from one- to multidimensional setting of the subdiffusive B–S model is at some points not straightforward. Property of lack of arbitrage depends on the parameters of the volatility matrix. Also, the martingale measure is not unique, its choice has crucial influence on the price.

Let us remind that the celebrated B–S formula was derived by solving certain heat equation, so the motivation came clearly from physics. We expect that the similar process will be observed in financial engineering. Fractional operators, which are successfully used in statistical physics in the description of anomalous fractional dynamics will find important applications also in finance.

The first chapter of this work includes the basic knowledge about the options, classical one-dimensional and multidimensional B–S models and such characteristics of the financial market as lack of arbitrage and market completeness.

The second chapter introduces the concept of multidimensional B–S model, generalized to the subdiffusive case. At the beginning
the inverse stable subordinator is defined and its basic characteristics are given. The subdiffusive multidimensional GBM is introduced, i.e. the subordinated process used to model the prices of the underlying assets with specific periods of stagnation. The fractional multidimensional Fokker–Planck equation is derived, which gives the information about the dynamics of the probability density function (PDF) of the studied process [13,15,16]. In the following it is proved that the considered market is arbitrage-free and incomplete. It should be added that the lack of arbitrage for subdiffusive B-S model was also analyzed in [7]. The B-S formula of option pricing is derived, the price of basket call option depending on the stability parameter $\alpha$, maturity time and the strike price is approximated by the methods of numerical integration and Monte-Carlo simulations. The obtained price functions are compared with the classical case. We end the paper with concluding chapter. For the sake of clarity we moved the most technical proofs to the Appendix.

2. B-S: classical approach

2.1. Options

Let us recall the definition of an option. Option is a financial instrument such that its buyer (owner) has the right to buy (call option) or to sell (put option) an asset $Z(t)$ at some prespecified maturity time $T$ for a prespecified strike price $K$. The underlying asset is usually a stock but some other assets are also possible. A European option, in contrary to the American option, can be exercised at the maturity time only, whereas it is possible to exercise American option at any time up to expiration [1.17].

The payoffs of the European call option and the European put option are equal to $(Z(T) - K)_+$ and $(K - Z(T))_+$, respectively. Here $(x)_+ = \max(x, 0)$. There is a relationship between price of the put option $P$ and the price of a call option $C$. This relationship is called put-call parity [17]:

$$C(0) - Ke^{-rT} = P - C.$$  \hspace{1cm} (1)

Here $r \geq 0$ is the interest rate. Further on, we will assume for simplicity that $r = 0$.

Let us now introduce the concept of a basket option, which is defined as financial instrument, where the underlying asset is a portfolio of various assets $Z^{(i)}(t), i = 1, \ldots, m$, for instance, single stocks. Basket option among others belongs to the class of exotic options. The basket option implies higher cost-effectiveness than a simple collection of single options, as it is a diversification instrument which also takes into account the interdependences between diverse risk factors. As an example, there might be a negative correlation between stocks in the “basket”, strong enough to significantly reduce the total risk or even make it disappear [17]. Basket option, exercised in European call style, has the following payoff

$$(\sum_{i=1}^{m} \omega_i Z^{(i)}(T))_+,$$  \hspace{1cm} (2)

where $\{\omega_i\}_{i=1}^{m}$ are constant weights, such that $\sum_{i=1}^{m} \omega_i = 1$. Here $\text{constant } K > 0$ is the strike price.

Trading of option contracts has had a long historical life. However, the prevalence of the option market increased rapidly in 1973 when options regulations became standardized and transactions on the stock options were performed on the Chicago Board Options Exchange. Simultaneously in 1973 Black and Scholes [1] and Merton [14] published their celebrated papers. These papers had a great impact to the financial market and arouse huge academic interest, setting up the key principles of arbitrage option pricing.

While the fair prices of European options can be found using the classical B-S formula, to find the price of a basket option one needs to use the multidimensional models.

2.2. Classical B-S model

Over the last forty years the B-S [2,14,17,19] model is being a powerful tool for pricing derivatives. One may define it as a mathematical model that allows to simulate the prices of financial instruments, such as stocks, as well as derive the fair prices of certain financial derivatives such as European call options.

Let us consider such a market, that its development up to time $T$ is defined on the probability space $(\Omega, F, P)$, where $\Omega$ is the sample space, $F$ is a set of all events and possible statements about the prices on the market and $P$ is the usual probability measure. The price of an asset $Z_i$ in classical B-S model is assumed to follow GBM given by

$$dz_i = \left(\mu + \frac{1}{2}\sigma^2\right)Z_i dt + Z_i \sigma dW_i, \quad Z_0 = z_0$$  \hspace{1cm} (3)

or equivalently

$$Z_i = Z_0 \exp\{\mu t + \sigma W_i\}. \hspace{1cm} (4)$$

Here, $W_i$ is the standard Brownian motion with respect to $P$, $\sigma > 0$ is the diffusion (volatility) parameter and $\mu \in \mathbb{R}$ is the drift.

The celebrated B-S price of European call option equals [11]:

$$C_{B-S}(Z_0, K, T, \sigma) = Z_0 \Phi(d_1) - K \Phi(d_2), \hspace{1cm} (5)$$

with

$$d_1 = \frac{\log\frac{Z_0}{K} + \frac{1}{2}\sigma^2 T}{\sigma \sqrt{T}}, \hspace{1cm} \text{and} \hspace{1cm} d_2 = d_1 - \sigma \sqrt{T}.$$

Here $\Phi$ is the distribution function of Gaussian distribution with mean zero and variance equal to one. This fair price was originally derived by Black and Scholes by solving the well-known in physics heat equation.

By the mentioned put-call parity we have for the price of put option

$$P_{B-S}(Z_0, K, T, \sigma) = C_{B-S}(Z_0, K, T, \sigma) + K - Z_0. \hspace{1cm} (6)$$

The B-S model was certainly a break-through in the option pricing apparatus. However, it has number of limiting assumptions that do not reflect the real market conditions and behavior of assets’ prices. This gives a fruitful area for academic purposes; already existing and ongoing researches are aimed to possibly relax these assumptions.

2.3. Multidimensional B-S model

The classical B-S model can be generalized to the multidimensional case, i.e. the number of assets $m > 1$ and the asset prices $Z(t) = (Z_t^{(1)}, \ldots, Z_t^{(m)})$ follow multidimensional GBM as

$$dz_t^{(i)} = \left(\mu_i + \frac{1}{2} \sum_{j=1}^{n} \sigma_{ij}^2\right)Z_t^{(i)} dt + \sum_{j=1}^{n} \sigma_{ij} dW_t^{(j)}, \quad Z_0^{(i)} = Z_{0i},$$  \hspace{1cm} (7)

or equivalently

$$Z_t^{(i)} = Z_0^{(i)} \exp\left\{\mu_i t + \sum_{j=1}^{n} \sigma_{ij} W_t^{(j)}\right\}, \quad i = 1, \ldots, m,$$

where $W(t) = (W_t^{(1)}, \ldots, W_t^{(m)})$ is $n$-dimensional Brownian motion with respect to $P$, $\{\sigma_{ij}\}_{m \times n}, \sigma_{ij} \geq 0$, is non-singular volatility matrix and $\{\mu_1, \ldots, \mu_m\}$ is a drift vector.

Multidimensional B-S model allows to find the fair price of basket options defined in (2). Unfortunately, there is no explicit B-S formula for such purpose. The price of a basket option in the classical multidimensional B-S model can be found using Gentile’s approximation by geometric average [6,17]. It is given by

$$C_{B-S} = \left(\sum_{i=1}^{m} \omega_i Z_0^{(i)}\right)(c\Phi(l_1(T)) - (R + c - 1)\Phi(l_2(T))), \hspace{1cm} (8)$$

Please cite this article as: G. Karipova, M. Magdziarz, Pricing of basket options in subdiffusive fractional Black-Scholes model, Chaos, Solitons and Fractals (2017), http://dx.doi.org/10.1016/j.chaos.2017.05.013
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