Marked Hawkes process modeling of price dynamics and volatility estimation

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\textbf{ABSTRACT}

A simple Hawkes model have been developed for the price tick structure dynamics incorporating market microstructure noise and trade clustering. In this paper, the model is extended with random mark to deal with more realistic price tick structures of equities. We examine the impact of jump in price dynamics to the future movements and dependency between the jump sizes and ground intensities. We also derive the volatility formula based on stochastic and statistical methods and compare with realized volatility in simulation and empirical studies. The marked Hawkes model is useful to estimate the intraday volatility similarly in the case of simple Hawkes model.

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\textbf{1. Introduction}

In this paper, the tick dynamics of stock prices observed at ultra-high-frequency level are modeled based on the symmetric marked Hawkes process and the empirical properties of the price dynamics are examined. The simple self and mutually excited Hawkes model for the price dynamics with the unit jump size incorporates the stylized facts of the ultra-high-frequency financial data, such as market microstructure noise and order clustering. On the other hand, random size jumps, i.e., not a constant jump, as in the simple Hawkes model, are observed in the tick structure of equity markets, particularly when there is a high ratio between the stock price and minimum tick size. By combining the Hawkes model with a mark structure, which has additional information for each event, more realistic model of the tick price dynamics is proposed to deal with the random size jumps.

Recent studies on (ultra)-high-frequency data and the market microstructure have been developed in several ways. The volume of literature on the financial theory of the market microstructure and limit order book (Roşu, 2009), and the role of algorithmic trading at a high-frequency rate (Chaboud et al., 2014; Foucault, 2012; Hoffmann, 2014) is increasing. A number of

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studies focused on the reduced form or stochastic modeling of the limit order dynamics and order executions; the reader may refer to Lo et al. (2002), Cont et al. (2010), Malo and Pennanen (2012), Cont and De Larrard (2013), Abergel and Jedidi (2013).

The statistical property of the ultra-high-frequency data is also an important subject because they exhibit the distinctive characteristics from the macro price dynamics. For example, care should be taken when applying the typical statistical methods to ultra-high-frequency data, and when computing the realized volatility (Andersen et al., 2003) due to microstructure noise, which refers to the mean reverting properties of the price processes at high frequency level. Previous studies (Alt-Sahalia et al., 2005, 2011; Hansen and Lunde, 2006; Zhang et al., 2005) measured the volatility of the return in the presence of market microstructure noise. Huth and Abergel (2014) examined the lead/lag relationship between asset prices and showed that there are significant cross correlations in the futures/stock at the high-frequency contrast with the daily data cases where cross correlations are negligible. The lead/lag relationship among the international index futures of different countries were also observed by Alsayed and McGroarty (2014). For the semi-Markov model with price jumps to explain the microstructure noise, consult Fodra and Pham (2015).

The financial asset price time series at the ultra-high-frequency level exhibits several autocorrelations that are not observed on a daily basis. Under the tick structure with a minimum tick size of price variation, the price dynamics is a pure jump process that consists of up jumps and down jumps. First, the frequency of up movements tends to increase with increasing frequency of the past down movements and vice versa. This causes a mean reverting property in the price dynamics, even though the correlations last for less than a few seconds. Second, there are also autocorrelations between the movements of the same direction. This causes volatility clustering that is different from the clustering on the macro level, which is typically modeled by GARCH (Bollerslev, 1986) or the stochastic volatility model (Heston, 1993), because the clustering properties in the tick structure last for only a few seconds. The durations of the autocorrelations are much shorter than those of the autocorrelation observed on a daily level.

These properties are well incorporated into the Hawkes model, which belongs to the class of point processes and is introduced by Hawkes (1971). Therefore, there has been an increase in the related work of modeling price dynamics based on the Hawkes process. The bivariate Hawkes process was introduced to model the buy and sell order arrivals and the impact of the orders on future prices was examined (Hewlett, 2006). The generalized Hawkes models was used to study the dependence between the occurrence of time trades and changes to the mid quote as well as the dependences between trading days (Bowsher, 2007). Large (2007) examined the market resilience after trades using the limit order book data and mutually excited Hawkes models.

Bacry et al. (2012) explained the non-parametric estimation method for the symmetric Hawkes process based on high-frequency futures data. Based on the mutually exciting Hawkes process, Bacry et al. (2013) suggested the mathematical framework that incorporates the market microstructure noise and the Epps effect, which is the correlation between the returns of two different assets at high sampling frequency. The trade clustering properties of the price dynamics on the micro level was well incorporated by the self-excited Hawkes process (Da Fonseca and Zaatour, 2014). Da Fonseca and Zaatour (2015).

A multivariate Hawkes process was introduced to model the up and down price movements and buy and sell orders to explain the stylized facts of the market impact and microstructure (Bacry and Muzy, 2014). Zheng et al. (2014) suggested a multivariate Hawkes process to describe the dynamics of the bid and ask prices. Lee and Seo (2014) focused on the daily and intraday volatility estimation based on the symmetric Hawkes process and compared the result with the realized volatility. For more about the kernel estimation in the Hawkes model, consult Bacry et al. (2016) and for the correlation and lead-lag relationship in a multi-asset model using the Hawkes process, consult Da Fonseca and Zaatour (2016).

The previous studies focused mainly on the simple point process model, where the jump size is constant. In the present study, the existing simple Hawkes model was extended to the marked Hawkes model to handle more realistic price movements in stock markets where the random size jumps (marks) are introduced. A marked point process was introduced based on the ACM-ACD model, where points are the transaction time and the marks are information on the transaction (Russell and Engle, 2005). The Hawkes process was adopted to explain the aftermath effects of the marks, which is more convenient for calculating the useful formula. The future effects of the marks depend on the absolute sizes of the marks and hence a linear impact function is introduced to deal with the future impact of the mark. Our empirical study shows that the estimates of the slope parameter of the impact function are significant positive values in stock markets. This suggests that the larger marks tend to magnify the future intensities more than the smaller marks. For the distribution of the mark, a specific distribution is not assumed in this paper but the empirical distribution is used for estimations and volatility calculation. Our model is not limited to the independent mark as the empirical studies show the intensity dependent mark distribution.

The remainder of the paper is organized as follows. In Section 2, the marked Hawkes model is proposed to describe the tick price dynamics of equities. Section 3 and Section 4 present the simulation results and empirical studies, respectively. Section 5 concludes the paper. The mathematical proofs are reported in the Appendix.

2. Symmetric marked Hawkes model

2.1. Marked point process

This subsection introduces the basic concepts of marked point processes. The mathematical framework is in line with Daley and Vere-Jones (2003). With the given complete separable metric state space, $\mathcal{X}$, a point process is a measure to count
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