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Portfolio selection with mental accounts and delegation

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ABSTRACT

Das et al. (2010) develop an elegant framework where an investor selects portfolios within mental accounts but ends up holding an aggregate portfolio on the mean–variance frontier. This investor directly allocates the wealth in each account among available assets. In practice, however, investors often delegate the task of allocating wealth among assets to portfolio managers who seek to beat certain benchmarks. Accordingly, we extend their framework to the case where the investor allocates the wealth in each account among portfolio managers. Our contribution is threefold. First, we provide an analytical characterization of the existence and composition of the optimal portfolios within accounts and the aggregate portfolio. Second, we present conditions under which such portfolios are not on the mean–variance frontier, and conditions under which they are. Third, we show that the aforementioned analytical characterization is also applicable within the framework of Das et al. and thus improves upon their numerical approach.

1. Introduction

In a recent paper, Das et al. (2010) develop an elegant framework that integrates features of the behavioral model of Shefrin and Statman (2000) with the mean–variance model of Markowitz (1952).[1] Consistent with Shefrin and Statman, the investor in this framework divides his or her wealth among mental accounts (hereafter, 'accounts') such as retirement, education, and bequest accounts.[2] Within any given account, the investor selects the portfolio with maximum expected return subject to a constraint that captures his or her goals for the account. This constraint precludes the probability that the account's return is less than or equal to some threshold return from being above some threshold probability. Consistent with Markowitz, the investor's optimal portfolios within accounts all lie on the mean–variance frontier. Hence, the investor's aggregate portfolio resulting from these portfolios (hereafter, 'aggregate portfolio') also lies on it.[3]

This consistency with Markowitz's model is derived under the assumption that in each account the investor allocates his or her wealth among the same set of available assets. In practice, however, investors often delegate the task of allocating wealth among these assets to portfolio managers (hereafter, 'managers') as Roll (1992) emphasizes.[4] Of particular interest is the question of whether the consistency with Markowitz's model breaks down if investors delegate such a task. Accordingly, our paper examines this question.

In doing so, our paper develops a portfolio selection model with accounts and delegation. Like Das et al., we consider an investor who divides his or her wealth among accounts. Unlike Das et al., however, the investor allocates the wealth within each account among managers. More specifically, for each account, the investor selects an allocation among managers with maximum expected return subject to a constraint that captures his or her goals for that account. As in Das et al., this constraint precludes the probability that the account's return is less than or equal to some threshold

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[1] As Das et al. note, this framework is also consistent with Telser (1955). Das and Statman (2009) expand upon Das et al. by examining the optimal portfolios of investors with mental accounts when derivative securities are available.

[2] For an introduction to mental accounting, see, e.g., Thaler (1985, 1999) and Nofsinger (2011, Chap. 6 and 7). Recent empirical support for mental accounting in 401(k) plans can be found in Choi et al. (2009).

[3] Das et al. assume that short sales are allowed; when they are disallowed, the aggregate portfolio may not lie on the mean–variance frontier. However, Das et al. explore the no short sales case by using an example and find that such a portfolio lies close to it.

[4] Other papers that recognize the importance of portfolio delegation include, for example, Brennan (1993), Admati and Pfleiderer (1997), Gómez and Zapatero (2003) and Cornell and Roll (2005).

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return from being above some threshold probability. Also, the values of the threshold return and probability possibly depend on the account. Like Roll (1992), we assume that managers select portfolios with minimum tracking error variance (TEV) for some expected tracking error over their respective benchmarks.[5] For generality, we allow different managers to have different benchmarks and TEV aversion coefficients.

The motivation for our model can thus be seen by combining two literatures. The first literature involves behavioral portfolio theory. As noted earlier, our model incorporates the idea that investors view their aggregate portfolios as collections of accounts with different goals. The second literature involves delegated portfolio management. As noted earlier, our model incorporates the idea that investors allocate their wealth to managers who seek to minimize TEV while attempting to beat their respective benchmarks.[6]

Like Das et al., we begin by assuming that short sales are allowed. When managers' optimal portfolios do not span the mean–variance frontier, we obtain three main results. First, we characterize the investor's optimal allocations among managers. The optimal allocation within an account exists if and only if the threshold probability is sufficiently low and the threshold return is sufficiently small. Further, this allocation is a weighted arithmetic average of two allocations whose composition does not depend on the account. However, their weights in the optimal allocation among managers depend on it.

Second, we show that the portfolio resulting from the optimal allocation (hereafter, 'optimal portfolio') within any given account is on the mean–variance frontier if and only if some weighted average of managers' benchmarks is also on it. In the calculation of this average, the weight of each benchmark is the fraction of the investor's wealth in the account that is allocated to the manager who tracks it. Since the condition that the weighted average of benchmarks lies on the mean–variance frontier does not typically hold, optimal portfolios within accounts are generally inconsistent with Markowitz's model.[7]

Third, we show that the aggregate portfolio is on the mean–variance frontier if and only if some weighted average of managers' benchmarks is also on it. In the calculation of this average, the weight of each benchmark is the fraction of the investor's total wealth that is allocated to the manager who tracks it. Again, since the condition that the weighted average of benchmarks lies on the mean–variance frontier does not typically hold, the aggregate portfolio is generally inconsistent with Markowitz's model. The resulting sub-optimality of the aggregate portfolio is, broadly speaking, in line with the empirical work of Bailey et al. (forthcoming). Indeed, they find that behaviorally-biased investors who buy mutual funds select them sub-optimally (e.g., prefer high-expense funds instead of low-expense funds).

The finding that portfolio delegation with mental accounting leads an investor to select optimal portfolios within accounts and an aggregate portfolio that generally lie away from the mean–variance frontier is in sharp contrast with the model of Das et al. where such portfolios all lie on it. However, this finding is driven by the assumption that investors delegate the task of allocating wealth among assets to managers, not by mental accounting *per se*. Since managers select portfolios that generally lie away from the mean–variance frontier, an investor ends up selecting optimal portfolios within accounts and an aggregate portfolio that generally also lie away from it.

Our results are in line with the model of Das et al. only in the case when managers' optimal portfolios span the mean–variance frontier. Indeed, in this case we show that optimal portfolios within accounts and the aggregate portfolio are on the mean–variance frontier and coincide with those in their model. Importantly, our approach in such a case improves upon theirs in two respects. First, while they develop an approach that provides a *semi-analytical* expression for optimal portfolios within accounts, we provide an *analytical* expression for such portfolios. Second, they *numerically* solve for the value of the threshold return at or below which there exist feasible portfolios, whereas we present an *analytical* condition under which these portfolios exist.

Like Das et al., we close by utilizing an example to analyze the case when short sales are disallowed since no analytical results are available. The results differ from those when short sales are allowed in two main respects. First, it may be optimal for the investor to allocate the wealth within any given account to a smaller number of managers. Second, the optimal portfolios within accounts and the aggregate portfolio may involve positions in a smaller number of assets.

Previous work recognizes that it might be sub-optimal for an investor to use an investment approach that involves decentralized managers. Sharpe (1981) provides objective functions for managers so that this sub-optimality is alleviated. Jorion (2003) shows that allocating wealth evenly among managers may still result in a portfolio that is overly risky from the perspective of the investor. Elton and Gruber (2004) provide conditions under which it is optimal for an investor to instruct managers to select portfolios that are proportional to the appraisal ratios of available assets.[8] van Binsbergen et al. (2008) examine how to optimally select a benchmark to reduce decentralization costs. Our paper differs from this work in two main respects. First, we consider an investor with multiple accounts. Second, we assume that the investor allocates his or her wealth in any given account by selecting the allocation among managers with maximum expected return subject to a constraint that captures his or her goals for such an account.

We proceed as follows. Section 2 describes the model, and characterizes both the optimal portfolios within accounts and the aggregate portfolio when short sales are allowed. Section 3 provides an example to illustrate these portfolios. Section 4 explores the case when short sales are disallowed, and Section 5 concludes. The Appendix contains proofs of our theoretical results.

2. The model

Let $N > 2$ be the number of assets. The $N \times 1$ vector of their expected returns is denoted by $\boldsymbol{\mu}$ where the nth entry represents

[5] A portfolio's *tracking error* is the difference between the returns on: (i) the portfolio and (ii) the benchmark.

[6] Fraser and Jennings (2006) find that managing an endowment by dividing it into multiple sub-portfolios with different goals might bring diversification benefits. They note that endowments often hold portfolios not involving assets that are: (1) overly risky from the perspective of endowment trustees; but (2) useful to diversify the endowments' aggregate portfolios. Next, they show that using multiple sub-portfolios can lead the endowment to hold an aggregate portfolio that involves a larger number of assets (including the useful assets mentioned earlier) and thus is closer to the mean–variance frontier. Hence, our model can also be motivated by this idea when the investor and his or her multiple accounts represent an endowment with multiple sub-portfolios having different goals. Importantly, however, our model notably differs from the one of Fraser and Jennings. In particular, the investor in our model allocates the wealth within each of his or her accounts among managers, whereas the investor in the model of Fraser and Jennings directly allocates it among assets. For related work on the performance of university endowment funds, see Brown et al. (2010) and Brown and Tiu (2010).

[7] An extensive literature recognizes that managers can have incentives to take actions that are sub-optimal from the perspective of investors. First, these incentives can be induced *explicitly* by compensation contracts that are based on the managers' performance relative to a benchmark; see, e.g., Admati and Pfleiderer (1997), Elton et al. (2003), Goetzmann et al. (2007) and Giambona and Golec (2009). Second, the incentives can be induced *implicitly* by the relationship between fund inflows and performance; see, e.g., Gruber (1996), Chevalier and Ellison (1997), Sensoy (2009) and Elton et al. (2010). Berk and Green (2004) examine the flow-performance relationship in rational markets.

[8] An asset's *appraisal ratio* is defined as the asset's alpha divided by its residual risk. Here, alpha and residual risk are determined relative to a factor model.