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Portfolio selection with mental accounts and background risk

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ABSTRACT

Das et al. (2010) develop a model where an investor divides his or her wealth among mental accounts with motives such as retirement and bequest. Nevertheless, the investor ends up selecting portfolios within mental accounts and an aggregate portfolio that lie on the mean–variance frontier. Importantly, they assume that the investor only faces portfolio risk. In practice, however, many individuals also face background risk. Accordingly, our paper expands upon theirs by considering the case where the investor faces background risk. Our contribution is threefold. First, we provide an analytical characterization of the existence and composition of the optimal portfolios within accounts and the aggregate portfolio. Second, we show that these portfolios lie away from the mean–variance frontier under fairly general conditions. Third, we find that the composition and location of such portfolios can differ notably from those of portfolios on the mean–variance frontier.

1. Introduction

Das et al. (2010) develop an appealing model that incorporates features of both behavioral and mean–variance models. Consistent with Shefrin and Statman (2000), Das et al. consider an investor who divides his or her wealth among mental accounts (hereafter, 'accounts') with motives such as retirement and bequest.[1] Within each account, the investor seeks to select the portfolio with maximum expected return subject to a constraint that reflects the account's motive. This constraint precludes the probability that the account's return is less than or equal to some threshold return from exceeding some threshold probability. Consistent with Markowitz (1952), optimal portfolios within accounts are on the mean–variance frontier.[2] Therefore, the corresponding aggregate portfolio is also on it.[3]

Importantly, Das et al. assume that the investor only faces portfolio risk. In practice, however, many individuals also face background risk (i.e., risk that is not fully insurable in financial markets). Sources of background risk include, for example, labor income and real estate. Due to the practical prominence of background risk, its recognition is of particular interest. Indeed, Das et al. suggest an extension of their results to the case where background risk is present. In this paper, we provide such an extension.

In doing so, we develop a portfolio selection model with accounts and background risk. Like Das et al., we consider an investor who divides his or her wealth among accounts. Unlike Das et al., however, we assume that the investor faces background risk in each account. For any given account, the investor seeks to select the portfolio that, taking into consideration background risk, maximizes the account's expected return subject to a constraint that reflects the account's motive. This constraint precludes the probability that, again taking into consideration background risk, the account's return is less than or equal to some threshold return from exceeding some threshold probability.

The motivation for our model can be seen by combining two literatures. The first literature involves behavioral portfolio theory. As noted earlier, our model incorporates the idea that investors view their aggregate portfolios as collections of portfolios within accounts. The second literature involves portfolio selection with background risk. As also noted earlier, our model incorporates the idea that investors face background risk.

We begin by characterizing the existence and composition of optimal portfolios within accounts. We show that the optimal portfolio within a given account exists if and only if the threshold probability is sufficiently low and the threshold return is sufficiently small. Moreover, we find that the composition of optimal portfolios within accounts can differ notably from those of portfolios on the mean–variance frontier. Hence, the former portfolios can lie considerably away from this frontier.

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[1] For an introduction to mental accounting, see, e.g., Thaler (1999) and Nofsinger (2011, Chapters 6 and 7).

[2] As Das et al. point out, their model is also consistent with Telser (1955). Das and Statman (2009) expand upon this model by examining optimal portfolios within accounts when derivatives are available.

[3] This result assumes that short sales are allowed. In the case where they are disallowed, Das et al. find that the aggregate portfolio lies close to the mean–variance frontier.

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Similarly, we next characterize the existence and composition of the aggregate portfolio. The aggregate portfolio exists if and only if the threshold probability of each account is sufficiently low and the threshold return of each account is sufficiently small. Furthermore, we find that the composition of the aggregate portfolio can differ notably from those of portfolios on the mean–variance frontier. Hence, the former portfolio can lie considerably away from this frontier.

The finding that the optimal portfolios within accounts and the aggregate portfolio generally lie away from the mean–variance frontier contrasts with the results of Das et al. who show that such portfolios lie on it. However, we should emphasize that our finding is driven by the presence of background risk, not by mental accounting. Indeed, an investor with a mean–variance objective function who faces background risk and has a single account optimally selects a portfolio that generally lies away from the mean–variance frontier (see, e.g., Baptista, 2008; Jiang et al., 2010). Hence, an investor who faces background risk and has multiple accounts ends up selecting optimal portfolios within accounts and an aggregate portfolio that generally also lie away from it.

There is an extensive literature recognizing the effect of background risk on portfolio selection. For example, some papers provide conditions on utility functions under which the presence of background risk makes investors less willing to bear other risks (see, e.g., Pratt and Zeckhauser, 1987; Kimball, 1993; Gollier and Pratt, 1996). Other papers examine the effect of background risk on the optimal portfolios of investors who use an expected utility model (see, e.g., Heaton and Lucas, 2000). There are also papers that investigate the effect of background risk on the optimal portfolios of investors who use a mean–variance model (see, e.g., Flavin and Yamashita, 2002; Baptista, 2008; Jiang et al., 2010). Our paper differs from this literature in two respects. First, while we consider an investor with multiple accounts, the literature considers an investor with a single account. Second, we assume that for each account the investor seeks to maximize the account's expected return subject to a constraint that reflects the account's motive, whereas the literature assumes that the investor maximizes either expected utility or a mean–variance objective function.

We proceed as follows. Section 2 describes the model, and characterizes the optimal portfolios within accounts and the aggregate portfolio when short sales are allowed. Section 3 provides an example to illustrate these portfolios. Section 4 examines the case when short sales are disallowed. Section 5 concludes.[4]

2. The model

Let $N > 2$ be the number of risky assets. The $N \times 1$ vector of their expected returns is given by $\boldsymbol{\mu}$ where the nth entry represents asset n's expected return. The $N \times N$ variance–covariance matrix of asset returns is given by $\boldsymbol{\Sigma}$ where the entry in row n_1 and column n_2 represents the covariance between the returns on assets n_1 and n_2. We assume that rank $(\boldsymbol{\Sigma}) = N$. Like Das et al., we assume that a risk-free asset is not available.

A portfolio is an $N \times 1$ vector $\boldsymbol{w}$ with $\boldsymbol{w}'\mathbf{1} = 1$, where $\mathbf{1}$ is the $N \times 1$ vector $[1 \ \cdots \ 1]'$.[5] The nth entry of $\boldsymbol{w}$ is the weight of asset n in portfolio $\boldsymbol{w}$. Here, a positive (negative) weight represents a long (short) position.[6] Let $r_{\boldsymbol{w}}$ denote portfolio $\boldsymbol{w}$'s random return. Note that portfolio $\boldsymbol{w}$'s expected return is $E[r_{\boldsymbol{w}}] \equiv \boldsymbol{w}'\boldsymbol{\mu}$, whereas its variance is $\sigma^2[r_{\boldsymbol{w}}] \equiv \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}$.

In accordance with mental accounting, we consider an investor who views his or her aggregate portfolio as a collection of portfolios within accounts. Hence, like Das et al., we assume that the investor divides his or her wealth among $M \geqslant 2$ accounts. The allocation of the investor's wealth among these accounts is given by the $M \times 1$ vector $\boldsymbol{y}$ where the mth entry represents the fraction of wealth in account m.[7]

Unlike Das et al., however, we assume that background risk is present. More specifically, we make two assumptions in regard to background risk. First, we assume that the investor faces it in all accounts.[8] Second, we assume that he or she faces possibly different background risks in different accounts.

It should be stressed that these two assumptions are plausible in practice. For brevity, consider an individual who divides his or her wealth between retirement and bequest accounts. In practice, retirement accounts often have exposures to the stocks of the companies where individuals are employed (see, e.g., Benartzi and Thaler, 2001). Importantly, retirement plan sponsors often restrict employees from eliminating at least part of these exposures (see, e.g., Poterba, 2003; Markowitz et al., 2010). Similarly, in practice, bequest accounts often have exposures to real estate (see, e.g., Raub, 2009). These exposures often involve properties whose values are difficult to insure (e.g., personal residences and investment real estate). Company stock and real estate exposures can thus be viewed as different background risks in different accounts. Hence, it is plausible to assume that the investor in our model: (1) faces background risk in all accounts (our first assumption); and (2) faces possibly different background risks in different accounts (our second assumption).

The $M \times 1$ vector of expected values of the background risks is given by $\boldsymbol{\upsilon}$ where the mth entry represents the expected value of account m's background risk. The $M \times M$ variance–covariance matrix of background risks is given by $\boldsymbol{\Omega}$ where the entry in row m_1 and column m_2 represents the covariance between the background risks of accounts m_1 and m_2. Hence, the variance of account m's background risk is denoted by Ω_{mm}.

The $N \times M$ matrix of covariances between asset returns and background risks is given by $\boldsymbol{\Psi}$ where the entry in row n and column m represents the covariance between asset n's return and account m's background risk. Therefore, the $N \times 1$ vector of covariances between asset returns and account m's background risk is denoted by $\boldsymbol{\Psi}_m$.

Fix any given account $m \in \{1, \ldots, M\}$. If the investor selects portfolio $\boldsymbol{w}$ within account m, then the account's random return is denoted by $r_{\boldsymbol{w},m}$.[9] Furthermore, the account's expected return and variance are given by $E[r_{\boldsymbol{w},m}] \equiv \boldsymbol{w}'\boldsymbol{\mu} + \upsilon_m$ and $\sigma^2[r_{\boldsymbol{w},m}] \equiv \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} + \Omega_{mm} + 2\boldsymbol{w}'\boldsymbol{\Psi}_m$, respectively.

The optimal portfolio within account m solves:

$$\max_{\boldsymbol{w} \in \mathbb{R}^N} \quad \boldsymbol{w}'\boldsymbol{\mu} + \upsilon_m \tag{1}$$

$$s.t. \ \boldsymbol{w}'\mathbf{1} = 1 \tag{2}$$

$$P[r_{\boldsymbol{w},m} \leqslant H_m] \leqslant \alpha_m, \tag{3}$$

where $H_m \in \mathbb{R}$ and $\alpha_m \in (0, 1/2)$ denote account m's threshold return and threshold probability, respectively. Consistent with mental accounting, we follow Das et al. in assuming that the investor selects the portfolio within any given account while ignoring the port-

[4] An Appendix containing proofs of the theoretical results in our paper can be downloaded at <http://home.gwu.edu/~alexbapt/JBF4Appendix.pdf>.

[5] Note that we use ' ′ ' to transpose a vector.

[6] Section 4 examines the case when short sales are disallowed.

[7] Following Das et al., we assume that $\boldsymbol{y}$ is exogenously determined. Note that allowing the investor to endogenously determine $\boldsymbol{y}$ might be inconsistent with the idea of having multiple accounts. Indeed, this idea breaks down if the investor allocates 100% of his or her wealth to a single account.

[8] Nevertheless, our model is more generally applicable when the investor is assumed to face background risk in $\underline{M}$ accounts where $0 < \underline{M} \leqslant M$.

[9] Like Das et al., we assume that the investor directly allocates the wealth in each account among available assets. Alexander and Baptista (2011) expand upon Das et al. by assuming that the investor allocates the wealth in each account among portfolio managers, which in turn allocate the wealth under management among available assets.