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# A survey of swarm intelligence for portfolio optimization: Algorithms and applications

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ABSTRACT

In portfolio optimization (PO), often, a risk measure is an objective to be minimized or an efficient frontier representing the best tradeoff between return and risk is sought. In order to overcome computational difficulties of this NP-hard problem, a growing number of researchers have adopted swarm intelligence (SI) methodologies to deal with PO. The main PO models are summarized, and the suggested SI methodologies are analyzed in depth by conducting a survey from the recent published literature. Hence, this study provides a review of the SI contributions to PO literature and identifies areas of opportunity for future research.

## 1. Introduction

Diversification of investments is a well-established practice used for reducing the total risk of investing for ages in the early history of portfolio theory [1]. A simple diversification strategy that spreads investment into different securities from various sectors, companies, businesses, locations and governments is commonly used. This strategy when applied alone may overlook the big picture since the attention is given on the individual securities rather than the relationship among the securities. Markowitz [2] proposed the mean-variance (M-V) model that brings a quantitative approach to portfolio management by using variance as a measure of economic risk for a desired efficient diversification of securities. Thus, in addition to the characteristics of individual securities, the relationships among all securities can also be considered. Although the use of variance as a risk measure [2,3] has been a touchstone in the history of portfolio theory, alternative risk measures such as Variance with skewness (VwS) [4], Value-at-Risk (VaR) [5], Conditional Value-at-Risk (CVaR) [6], Mean-Absolute Deviation (MAD) [7] and Minimax (MM) [8] portfolio optimization models have been proposed in the literature.

Limitations faced by real-life investors weaken the direct applicability of basic PO models, having a lack of specific constraints such as boundary constraints (BC) [9], cardinality constraints (CC) [10], transaction costs (TC) [11] and transaction lots (TL) [7]. Unconstrained portfolio optimization, a typical convex quadratic programming problem, can be efficiently solved by exact approaches such as linear and quadratic programming. However, as proven by Bienstock [10], adding practical restrictions such as cardinality constraints to the model, the problem is converted to mixed-integer quadratic programming which is dragged into NP-complete class of problems which limits the computational efficiency of exact solution approaches while the problem size increases. Therefore, researchers paid a particular attention on developing approximation methods such as heuristic/metaheuristic algorithms. Evolutionary Algorithms (EA) and Swarm Intelligence (SI) methodologies are two of the most preferred solution approaches for portfolio optimization. Metaxiotis and Liagkouras [12] presented a literature review of multi-objective EA while Kalayci et al. [13] presented a recent review of genetic algorithms for portfolio optimization. Unlike EA utilizing principles of natural selection, SI methodologies are inspired by the behaviors and self-organizing interaction among agents, such as foraging of ant and bee colonies, bird flocking or fish schooling. Some of the SI algorithms adopted for solving PO variants are listed as follows: particle swarm optimization (PSO) [14], ant colony optimization (ACO) [15], bacterial foraging optimization (BFO) [16], artificial bee colony (ABC) [17], cat swarm optimization (CSO) [18], firefly algorithm (FA) [19], invasive weed optimization (IWO) [20], bat algorithm (BA) [21] and fireworks algorithm (FA) [22].

According to the authors' knowledge, there is no study that specifically provides a comprehensive review of SI methodologies adopted for solving portfolio optimization problems. Therefore, considering the attracted attention of both academics and practitioners, it is time of interest to present how the SI for the portfolio optimization field has evolved and what its present state is. The objective is to present and gain an understanding of the current state of research in SI methodologies for

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portfolio optimization by providing a review of the available literature in the field to identify potential areas of concern.

The rest of the paper is structured as follows: Section 2 presents the models and constraints for portfolio optimization, Section 3 presents a quick peek into solution approaches for PO, Section 4 describes different SI algorithms and reviews applications on PO, Section 5 reviews problem specific application contexts, Section 6 discusses characteristics, advantages and adequacy of different methods and finally, Section 7 concludes the paper and provides future research issues and directions.

## 2. Models for portfolio optimization

In this section, we briefly present the PO models and additional constraints for realistic portfolio management. Table 1 presents a summary of different models for PO.

In the M-V model [2], the portfolio risk is measured by the variance of stock prices. In general, covariance matrix among individual stocks and expected return of stocks are estimated using the historical data. However, the M-V approach may lead to an insufficient prediction of portfolio given an asymmetric return distribution since M-V model assumes that expected returns have a symmetric multivariate normal distribution. Hence, Markowitz also suggested a model based on semi-variance (S-V) [2] which is more preferable for stock returns having an asymmetric distribution. In order to successfully capture the characteristics of the return distribution, another approach is to include skewness into the M-V model [15]. While positive skewness in portfolio returns may lead some reduction in downside risk which is favorable to investors, the coefficient of skewness is affected by returns greater or less than the mean return achieving portfolios with similar skewness but quite different downside behavior.

The M-V model consists of a quadratic objective function and linear constraints. While Markowitz [2] is said to be the father of modern portfolio theory, the M-V model has been criticized because of its structure and therefore, alternative risk measures such as VaR, CVaR, MAD and MM have been proposed in the literature. In order to overcome arising computational difficulties due to the quadratic structure of M-V model, Konno and Yamazaki [7] proposed MAD as an alternative risk measure that employ the mean of the absolute deviations of the portfolio return in all periods as the risk measure. The portfolio return is devised from the corresponding historical returns of the stocks. The risk in this model is measured by absolute deviation of the assets rate of return. Therefore, calculating the covariance matrix is unnecessary with the model having less computational cost and is very easy to update the model when new data are added. Furthermore, its risk function can be transformed into parametric linear programming, and thus portfolio optimization implementation is simplified. However, Simaan [23] reports that, although estimation error is more severe in small samples and for investors with high risk tolerance in both M-V and MAD models, M-V model provides lower estimation risk in small samples and for investors with a low risk tolerance.

Another risk measure is MM, proposed by Young [8]; it is based on game theory, uses minimum return rather than variance as a measure of risk to avoid the logical problems of a quadratic utility function implied by M-V portfolio selection rules. In MM model, portfolio risk is measured by the minimum portfolio return over all periods. Thus, in contrast to M-V and MAD models, in MM model, the risk measure is asymmetric which is claimed to be more appropriate for skewed return distributions [8].

CVaR [6], also known as Mean Excess Loss, Mean Shortfall, or Tail VaR, is proposed to approximate the joint density function by a number of scenarios to obtain a linear model. VaR [5] describing the quantile of the projected distribution of gains and losses over the target horizon measures the worst loss (lowest return) a portfolio can potentially suffer [24]. However, VaR has undesirable mathematical characteristics such as a lack of subadditivity and convexity [6]. CVaR aiming to reduce the risk of high losses based on VaR representing an asymmetric risk measure that can be also used in return and risk analyses.

Fig. 1 demonstrates the distribution of considered models for portfolio optimization in SI literature where M-V model is highly dominant over other models.

**Table 1**
Models for portfolio optimization.

| Model | Proposed By | Structure | Year |
|---|---|---|---|
| Mean-variance (M-V) | Markowitz [2] | Quadratic | 1952 |
| Variance with skewness (VwS) | Samuelson [4] | Quadratic | 1958 |
| Semi-variance (S-V) | Markowitz [3] | Quadratic | 1959 |
| Mean- Absolute Deviation (MAD) | Konno and Yamazaki [7] | Linear | 1991 |
| Value-at-Risk (VaR) | Jorion [5] | Linear | 1997 |
| Minimax (MM) | Young [8] | Linear | 1998 |
| Conditional Value-at-Risk (CVaR) | Rockafellar and Uryasev [6] | Linear | 2000 |

### 2.1. Classification of portfolio optimization models

If the decision-making process and the future events in portfolio optimization are restricted to a one-time period, a single-period portfolio optimization (SPPO) model is under study. Although Markowitz [2,3] also considers multi-period models as a course of its nature where, if desired, the portfolio may be readjusted several times during the planning horizon, studying a sequence of decisions where transactions take place at discrete time points instead of once is analyzed in multi-period portfolio optimization (MPPO) model [25].

Fig. 2 demonstrates the distribution of model types for portfolio optimization in SI literature where SPPO model is highly dominant. On the other hand, there are a few attempts in SI literature [26–28] on solving multi-period decision making models in which future events are spread over several periods.

### 2.2. Additional constraints for realistic portfolio management

Limitations faced by real-life investors weaken the direct applicability of basic PO models, having a lack of specific constraints, where compulsory criteria are required, or easier implementation is desired.

Although unconstrained (UC) model, namely, suggests having no constraints in the model, the following limitations are present in the default unconstrained setting: the investment of all funds is ensured, and short sales are not allowed. The investment of all funds is verified by exactly restricting the sum of security proportions to one and short sales are excluded by utilizing non-negativity constraints [2]. In addition to default unconstrained settings, additional constraints such as boundary constraints (BC) [9], cardinality constraints (CC) [10], transaction costs (TC) [11] and transaction lots (TL) [7] are needed for a more realistic portfolio management.

Small proportions held in the portfolio have typically little impact on the performance and have weak liquidity and can be usually costly in terms of brokerage fees or monitoring costs. Therefore, in practice, a lower limit preventing the holding of a position if one does not invest in more than the minimal allowable position, called as min-buy or minimum transaction level, may be desired/enforced when clients buy, sell or revise stocks. An upper limit may also be used for flexibility. Therefore, boundary constraints (BC) are added as a constraint in mathematical formulations.

It is not possible for an investor/fund manager to purchase all the securities in the index according to their weights. Therefore, a common practice is to purchase just a subset of the entire set of securities for easier management of the portfolio. Therefore under such circumstances, on top of enforcing boundaries, cardinality constraints [10] are also needed to define an upper limit on the number of securities to be held in a portfolio.