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LGEM: A lattice Boltzmann economic model for income distribution and tax regulation

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ABSTRACT

In this paper, a new econophysics model based on a lattice Boltzmann automata is presented. This model represents economic agents (people, countries...) as particles of a gas moving on a 2D lattice and interacting with each other. Economic transactions are modeled by particle-to-particle interactions in which money is conserved. If only particular transactions are considered (free market), the money distribution quickly converges to a Boltzmann–Gibbs distribution. But the model also introduces a third step of global income distribution that can be used for exploring tax regulation strategies. The model is presented, and some examples of income distribution are given. One of the most interesting features of the model is the fact that it is completely discrete, and it can be exactly implemented on any computational resource, leading to very fast, yet powerful simulations, especially when parallelization resources are available. Some results of these simulations, as well as performance data, are given.

1. Introduction

Econophysics is an interdisciplinary research field which applies statistical physics methods for solving problems in economics and finance. The term "econophysics" was first introduced by the theoretical physicist Eugene Stanley at the conference *Dynamics of Complex Systems* held in Calcutta in 1995, as an analogy with similar terms such as "astrophysics" or "biophysics", which describe applications of physics to different fields [1]. This novel discipline uses mathematical methods developed in statistical physics to study statistical properties of complex economic systems consisting of a large number of humans, and it can be considered as a branch of applied theory of probabilities [2]. In this sense, econophysics has much common ground with agent-based modeling and simulation, as it studies mathematical models of a large number of interacting economic agents [3]. Econophysics distances from the classical approach of economics, mainly representative-agent based, which ignores statistical and heterogeneous aspects of the economy.

The main trend in econophysics can be reduced to a process consisting of 4 steps: (1) propose a micro-model that captures the essence of the economic interaction between two agents, (2) identify conserved quantities in the model, (3) integrate the micro-model to the macroscopic scale when millions of agents participate in the economy, and (4) get results and compare it with real data.

In this paper, we are going to follow a different approach, inspired by a new trend in statistical mechanics, not previously applied to the economy field: lattice gas automata.

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2. Statistical physics applied to economy

Statistical mechanics started in the XIX century, in particular, Maxwell and Boltzmann developed the probability distribution of molecules in a gas, while Boltzmann and Gibbs obtained the general probability distribution of states as a function of the energy [4,5]. As time passed by, the social origin of statistical physics was mainly forgotten, although some early attempts took place along the XX century, from scientists like Ettore Majorama or Elliott Montroll [6]. But it was not until late 1990s that those attempts led to the emergence of econophysics. One of the first concerns of modern econophysics is money and wealth distribution. This subject was addressed by Dragulescu and Yakovenko [7] and, independently of them, by Chakraborti and Chakrabarti [8].

Those works referred to the fundamental law of statistical mechanics, that is, the Boltzmann–Gibbs distribution, which states that the probability of finding a physical system in an estate with the energy ϵ is given by:

$$P(\epsilon) \propto e^{-\beta\epsilon} \tag{1}$$

where β is a parameter inversely related to the temperature of the system. The average value of any physical quantity x can be obtained as:

$$\langle x \rangle = \frac{\sum_i x_i e^{-\beta\epsilon_i}}{\sum_i e^{-\beta\epsilon_i}} \tag{2}$$

where i runs over all the possible states of the system. The analytical derivation of (1) only needs two conditions to be applied to a system: (1) statistical character of the system itself, and (2) conservation of energy ϵ [9]. However, Ludwig Boltzmann found (1) to be a particular consequence of a more general principle, namely the second law of thermodynamics, which uses the concept of entropy.

In systems with many particles, the entropy per particle S can be written as:

$$S = -\sum_i P_i \ln P_i. \tag{3}$$

The second law of thermodynamics states that every system tends to the state of maximum entropy. But our system presents the important restriction that the total energy is a constant and, consequently, the average energy is also a constant.

$$\sum_i N_i\epsilon_i = E \longrightarrow \sum_i P_i\epsilon_i = \langle \epsilon \rangle. \tag{4}$$

In the previous expression, N_i is the number of particles with energy ϵ_i, $N_i = P_i \cdot N$, and E is the total energy of the system, $E = \langle \epsilon \rangle \cdot N$. N is the total number or particles.

This problem can be easily solved using the method of Lagrange multipliers, by maximizing the functional:

$$S - \alpha\left[\sum_i P_i - 1\right] - \beta\left[\sum_i P_i\epsilon_i - \langle \epsilon \rangle\right] \tag{5}$$

where α and β are Lagrange multipliers (in this expression, we have also introduced the normalization constraint). Variating the functional, we obtain:

$$\begin{gathered} \frac{\partial}{\partial P_j}\left\{-\sum_i P_i \ln P_i - \alpha\left[\sum_i P_i - 1\right] - \beta\left[\sum_i P_i\epsilon_i - \langle \epsilon \rangle\right]\right\} = 0 \\ \ln P_j = -1 - \alpha - \beta\epsilon_j. \end{gathered} \tag{6}$$

And the expression for the probability is:

$$P_j = e^{-1-\alpha-\beta\epsilon_j}. \tag{7}$$

Lagrange multipliers can be solved using system constraints. We can introduce the partition function $Z = e^{1+\alpha}$. The normalization constraint leads us to:

$$Z = \sum_j e^{-\beta\epsilon_j}. \tag{8}$$

And, what is more important, it offers to the distribution the form:

$$P_j = \frac{e^{-\beta\epsilon_j}}{Z} \tag{9}$$