Random fuzzy mean-absolute deviation models for portfolio optimization problem with hybrid uncertainty

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Abstract
Absolute deviation is a commonly used risk measure, which has attracted more attentions in portfolio optimization. The existing mean-absolute deviation models are devoted to either stochastic portfolio optimization or fuzzy one. However, practical investment decision problems often involve the mixture of randomness and fuzziness such as stochastic returns with fuzzy information. Thus it is necessary to model portfolio selection problem in such a hybrid uncertain environment. In this paper, we employ random fuzzy variables to describe the stochastic return on individual security with ambiguous information. We first define the absolute deviation of random fuzzy variable and then employ it as risk measure to formulate mean-absolute deviation portfolio optimization models. To find the optimal portfolio, we design random fuzzy simulation and simulation-based genetic algorithm to solve the proposed models. Finally, a numerical example for synthetic data is presented to illustrate the validity of the method.

1. Introduction
The traditional portfolio optimization method initiated by Markowitz [25] employs variance as risk measure and have become a cornerstone of modern portfolio theory. However, due to the computational difficulty associated with solving a large-scale mean-variance model, several authors tried to explore alternative risk measures to replace variance. Among others, absolute deviation introduced by Konno and Yamazaki [11] is one of the most commonly used risk measures in portfolio theory. In their paper, Konno and Yamazaki converted the corresponding mean-absolute deviation (MAD) model to a linear programming one, which removes most of the difficulties associated with mean-variance model. Since then, MAD model has attracted more and more attentions both in theory and in application. Konno et al. [12] first extended the model by introducing skewness, i.e., the third moment, Konno and Wijayanayake [13] discussed the model under transaction costs, and Chang [1] employed goal programming approach to further simplify MAD model. As applications, Zenios and Kang [39] used MAD model to manage portfolios of mortgage-backed securities and showed that it was superior over traditional portfolio optimization approaches. Konno and Koshizuka [14] further showed that the MAD model is more compatible with the fundamental principle of rational decision-making. Absolute deviation as risk measure is also applied to other types of portfolio optimization such as multi-period case (Yu et al. [36], Yu and Wang [37]) and robust model (Moon and Yao [26]).

All the above works implicitly assume that security returns are random variables and employ stochastic programming approach to formulate investment decision models. These models are valid if enough data are available for the practical problem. However, there are sometimes lack of data such as in an emerging financial market. The parameters or probability distributions of random returns are difficult or impossible to be accurately estimated. However, for this situation, the corresponding quantities may be provided by the experts based on their past information and subjective belief. In other words, security returns may be considered as fuzzy variables instead of random variables when lack of data. This is so called fuzzy portfolio optimization methods which have been presented and widely investigated in the literature such as Huang [8], Qin et al. [27], Tanaka and Guo [32], Wang and Zhu [34], Zhang and Wang [40]. Among others, Qin et al. [28] first introduced MAD model for fuzzy returns based on credibility theory and Chen et al. [3] further studied the properties of absolute deviation of fuzzy variable and extended MAD models. In addition, Liu [21] also considered interval-valued returns and proposed MAD models for that case, which can be regarded as the special case of fuzzy portfolio optimization since an interval is equivalent to an equipossible fuzzy variable.

Actually, the investors maybe encounter varieties of uncertainties such as randomness, fuzziness, or hybrid uncertainty consisting...
of randomness and fuzziness when handling the practical portfolio optimization problem. For example, the probability distributions of
security returns may be partially known. Therefore, the uncertain
parameters may be estimated by experts on the basis of available
data instead of only their belief, which implies that security returns
may be characterized by random variables with fuzzy information.

More specifically, for each scenario, the return on each security is
random variable instead of a crisp number. In such a way, the ran-
domness and fuzziness associated the problem are simultaneously
dealt with by using random fuzzy variable [18,23,43]. In fact, some
authors have applied random fuzzy variable to solve various prac-
tical problems. For example, random fuzzy renewal process was first
presented by Zhao et al. [42], which was further extended to ran-
dom fuzzy renewal reward process [30], random fuzzy alternating
renewal process [31], random fuzzy delayed renewal process [15],
respectively. It was also applied to project selection [9], project
scheduling [10], shock model [22], transportation planning [38],
DEA model [33], location problem [35] and other optimization
problems. In particular, several authors have also applied random
fuzzy variable to successfully model portfolio optimization with
hybrid uncertainty such as Hasuike et al. [4], Huang [7,8] and Qin
et al. [29].

To the best of the author’s knowledge, there is no research
devoted to applying absolute deviation to measure risk in hybrid
uncertain environment. This paper will attempt to fill this gap
and extend Konno and Yamasaki’s work to model the portfolio
optimization with random fuzzy returns. Before establishing the
models, we first give the definition of absolute deviation for random
fuzzy variable and discuss its mathematical properties. Then the
general form of random fuzzy MAD model is formulated to capture
the essential idea of trading off return and risk. Several variants of
models are offered to be consistent with investors’ preferences. Due
to the inherent complexity associated with random fuzzy variable,
it is difficult or even impossible to transform the proposed mod-
els into crisp ones, which implies that traditional algorithms will
fail to solve them. As a commonly used heuristic algorithm, genetic
algorithm offer the advantage of solving portfolio optimization and
it has been used by several researchers such as Chang et al. [2], Lin
and Liu [17], Huang [7] in this area. In view of this, the paper uses
genetic algorithm with integration of random fuzzy simulation to
solve the proposed MAD models, in which random fuzzy simul-
aton is designed to approximately calculate the values of mean and
absolute deviation of the return on a portfolio.

The rest of the paper is organized as follows. Section 2 reviews
some preliminaries about fuzzy variable and random fuzzy variable.
Section 3 proposes the concept of absolute deviation of random
fuzzy variable and discusses its several properties. Section 4 formu-
lates several random fuzzy MAD models for portfolio optimization
problem. Solution procedures are given in Section 5 which includes
random fuzzy simulation and key procedures of genetic algorithm.
A numerical example is presented and solved for synthetic ran-
dom fuzzy data in Section 6. Finally, some conclusions are listed in
Section 7.

2. Preliminaries

Let be a nonempty set, and be the power set of . A set function is called credibility measure [16,20] if it satisfies:
(1) (Normality) ; (2) (Monotonicity) whenever ; (3) (Self-Duality) ; (4) (Maximality) for any sequence such that with
such that . The triplet is called a credibility space.

A fuzzy variable is defined by Liu and Liu [20] as a measurable
function from a credibility space to the set of real numbers. Its membership function of fuzzy variable is derived from the credibility measure
by
(1)
The expected value of is defined by Liu and Liu [20] as

provided that at least of the two integrals is finite.

Example 1. Assume that is a simple fuzzy variable whose membership function is given by
(2)

A random fuzzy variable is defined by Liu [19] as a measurable function from a credibility space to the set of random variables. It is known that a crisp number may be regarded as a special random variable. In this sense, a fuzzy variable is a special case of random fuzzy variable. For each fixed and its expected value is a crisp number if the value exists. Obviously, is a function of a mapping from to the set of real numbers, which implies that is a fuzzy variable as changes.

Example 2. Let , , ..., be random variables, and , , ..., be random numbers in such that Then

is a random fuzzy variable which is called a simple random fuzzy variable.

Example 3. A random fuzzy variable is said to be normal if for each and is a normally distributed random variable with the following probability density function

where is a fuzzy variable defined on the credibility space. It is denoted by and the fuzziness of is characterized by fuzzy variable .

Definition 1 (Liu and Liu [24]). Let be a random fuzzy variable defined on the credibility space. The expected value of is defined as

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