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# Dynamics of money and income distributions

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**Abstract**

We study the model of interacting agents proposed by Chakraborti and Chakrabarti [Eur. Phys. J. B 17 (2000) 167] that allows agents to both save and exchange wealth. Closed equations for the wealth distribution are developed using a mean field approximation.

We show that when all agents have the same fixed savings propensity, subject to certain well-defined approximations defined in the text, these equations yield the conjecture proposed by Chakraborti and Chakrabarti [Eur. Phys. J. B 17 (2000) 167] for the form of the stationary agent wealth distribution.

If the savings propensity for the equations is chosen according to some random distribution, we show further that the wealth distribution for large values of wealth displays a Pareto-like power-law tail, i.e., $P(w) \sim w^{1+a}$. However, the value of $a$ for the model is exactly 1. Exact numerical simulations for the model illustrate how, as the savings distribution function narrows to zero, the wealth distribution changes from a Pareto form to an exponential function. Intermediate regions of wealth may be approximately described by a power law with $a > 1$. However, the value never reaches values of $\sim$1.6–1.7 that characterise empirical wealth data. This conclusion is not changed if three-body agent exchange processes are allowed. We conclude that other mechanisms are required if the model is to agree with empirical wealth data.


*Keywords:* Elastic and inelastic scattering; Kinetic theory; Classical statistical mechanics; Probability theory; Stochastic processes and statistics; Dynamics of social systems; Environmental studies

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## 1. Introduction

The distribution of wealth or income in society has been of great interest for many years. Italian economist Pareto [1] was the first to suggest, it followed a "natural law" where the higher end of the wealth distribution is described by power law, $P(w) \sim w^{-1-\alpha}$. Repeated empirical studies by Levy and Solomon [2], Dragulescu and Yakovenko [3], Reed and Hughes [4] and Aoyama et al. [5] show that the power-law tail exhibits a remarkable spatial and temporal stability and while the value of the exponent, $\alpha$, may vary slightly, it changes little from the value $\sim 1.5$.

Even though the collected data stem from different sources and can be incomplete because of difficulties in accessibility (poor conclusions from income data in Sweden [2], due to a too small number of wealth ranges in the data; total net capital of individual at death in the United States (US) reported to the Bureau of Census and the Inland Revenue for tax heritage purposes [3]; distributions of sizes of incomes, cities, Internet files, biological taxa, gene family and protein family frequencies [4]; and income distributions in the Japan [5]) the common conclusion which can be drawn is that the high end that exhibits the power law is characterised by several multiples or even tens of multiples of the average income/wealth (only 5% of population income-data in the US conforms to a power law and the power law for the yearly income data in the United Kingdom sets in only for $> 50\,\mathrm{k£}$ [3], income distributions in the Japan in 2000 exhibit power laws only for $> 5 \times 10^4$ thousands of Yen).

For around 100 years the tantalising Pareto law remained without explanation. The renewed interest by physicists and mathematicians in econo- and socio-physics has, however led to publication of a number of new papers on the topic in recent years (see [6] for an extensive literature review).

The fact that multiplicative power-law processes can lead to power-law distributions has been known for many years from studies as diverse as the frequency of words in text [7], economic growth [8], city populations [9], wealth distribution [10] and stochastic renewal processes [11].

In the analysis of these distributions Solomon et al. [12] have recently proposed the use of Generalised Lotka–Volterra (GLV) equation that combines a multiplicative random process with an auto-catalytic process. The latter redistributes a fraction of the total money to ensure the money possessed by an agent is never zero. This simulates in a simplistic way the effect of a tax. The model equations lead to a wealth distribution $P(w)$ of the form

$$P(w) \sim \frac{\mathrm{e}^{(1-\alpha)/w}}{w^{1+\alpha}} , \tag{1}$$

where and $\alpha - 1$ is a positive number that is a ratio of parameters of the model that are related to social security and some random investments, respectively. For large values of income $w$ this indeed exhibits a Pareto behaviour.

However, two issues arise. The first is that empirical studies of income distributions show that this function does not describe well the very low end of the income distribution which is essentially exponential [3]. The second relates to use