Convergence of VCG mechanism to ex-post budget balance in a model of land acquisition

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HIGHLIGHTS

- Land acquisition: a buyer is interested to buy plots from sellers located on a graph.
- To characterize ex-post budget surplus in VCG as number of sellers become large.
- We consider sequences of line and star graphs and more general sequences.
- Convergence depends on endpoints of the prior and number of critical sellers.

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ABSTRACT

We explore the effect of competition among a large number of sellers in a model of Land Acquisition. Sellers with one unit of land each are located at the nodes of a graph. Two sellers are contiguous if they are connected by an edge in the graph. The buyer realizes a positive value only if he can purchase plots that constitute a path of given minimal length. We characterize conditions on the prior for different graph structures under which the VCG mechanism almost surely results in an ex-post budget surplus as the number of sellers becomes large. Our results show that such convergence depends on (a) the endpoints of the support of valuations and (b) whether the number of critical sellers, i.e., sellers who lie in every feasible path, is preserved as new sellers are added to the underlying graph.

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1. Introduction

According to standard microeconomic theory, the market power of individual sellers declines as the number of sellers increases. A classic and extreme example of this is the comparison between standard monopoly and Bertrand duopoly: in the former, the market price is above the marginal cost, but in the latter, competition between two identical sellers drives market price down to marginal cost of production. In models of private information, the efficient outcome is often unattainable. A natural question is whether a second best mechanism converges to first best as the number of agents becomes large.

In this paper, we investigate the impact of increasing number of sellers in a multilateral exchange problem, viz., Land Acquisition. It refers to situations where a single buyer purchases a set of land plots from multiple landowners. Often plots are required to be contiguous so that large scale construction can take place. This problem has great relevance in many densely populated countries. Large scale construction often requires industry or the government to acquire vast areas of land that are inhabited and often cultivated by many people.\(^1\)

When prices acceptable to the buyer and the sellers are not public knowledge, the buyer has to negotiate with individual sellers who can respond by delaying strategically. This is commonly known as holdout. Often, the land acquisition exercise requires intervention from a third party in the form of arbitration, subsidies or coercion through constitutional means like eminent domain laws. Coercion by the State may lead to conflicts of social, political and economic significance.\(^2\)

This paper is a natural follow up of Sarkar (2016) who models the contiguity structure of land plots as graphs. In this model the buyer demands a path of a given minimal length (say, \(k\)) on a graph where each node represents a plot owned by a seller and every pair of physically adjacent plots is connected by an edge. It is natural

\(^1\) See Wikipedia (2014c) and Wikipedia (2014b); see Wikipedia (2014a) in the context of the United States.

\(^2\) See Chakravorty (2013) for an elaborate historical analysis of land acquisition in India; see Miceli (2011) for an analysis of eminent domain from a law and economics perspective.
that a buyer would prefer the acquired plots to be connected so that she has uninterrupted rights of passage. However, our analysis would go through when the buyer prefers subgraphs of size $k$ with any other connectivity structure as well. The paper characterizes priors for which Bayesian incentive compatible (BIC) mechanisms implement the first best. It also identifies the role of critical sellers who lie on all feasible paths. In particular, it is difficult to satisfy these conditions when the number of critical sellers is large.

Use of BIC mechanisms requires the mechanism designer to have precise information about the underlying priors. There has been emphasis on the construction of mechanisms that are robust with respect to such assumptions following a critique by Wilson (1987). A natural way to deal with this critique is to require mechanisms to be dominant strategy incentive compatible (DSIC). A mechanism is DSIC if no agent can ever gain in terms of ex-post payoffs by misreporting. The corresponding participation condition is ex-post individual rationality, or IR. A mechanism is IR if agents always get a positive payoff.

The VCG mechanism is DSIC and IR. However, it is not budget-balanced, or BB: the sum of VCG payments can be positive, negative or zero, depending on the profile and the prior. If the sum of payments is positive, a redistribution of the surplus will improve net welfare of agents. If the sum of payments is negative, the mechanism requires an outside subsidy. The VCG mechanism therefore, becomes approximately first-best in the limit if the sum of VCG payments is almost surely positive.

Sarkar (2016) uses a result due to Williams (1999) and Krishna and Perry (2000): a BB mechanism implements the first best if and only if the VCG mechanism results in ex-ante budget surplus. Consider the case where a buyer requires $k$ plots from $n$ sellers located on a complete graph with valuations distributed uniformly in $[0, 1]$: the VCG mechanism results in ex-ante budget surplus if buyer’s minimum possible valuation exceeds $\frac{nk}{n^2}$. Since $\frac{nk}{n^2} \to 0$ as $n \to \infty$, it becomes easier to satisfy the sufficient condition as the number of sellers increases.

This paper investigates the generalizability of the convergence noted in this example. Loosely speaking, we are asking the question: Is it easier to obtain first best in models of land acquisition with a large number of sellers? More specifically, consider a sequence of land acquisition problems in which the number of sellers progressively increase. We investigate the limit properties of the probability that VCG attains ex-post budget surplus over such a sequence. We are interested in influence of the underlying graph structure, number of items demanded by the buyer and the prior of valuations on such properties.

This question is of importance primarily because of two interrelated reasons: one, since VCG is a well-understood mechanism, it makes sense to investigate conditions that make it self-financing in the current context. The second reason is intimately related to the achievability of the first best as discussed in the preceding paragraph. First best mechanisms exist if VCG is self-financing in the ex-ante sense. Therefore, this paper also resolves the question whether first best mechanisms will exist when the number of sellers becomes large.

Note that the underlying graph may change depending on the way new sellers are added to an existing graph. We begin our study with some special cases where this issue can be easily handled. The first of these is a model where new sellers are added consecutively on a line. The second is a star graph where new sellers form additional edges with the seller located on the hub.

Let $k$ be the number of contiguous plots required by the buyer. Priors satisfy the Trade in the Limit or the TL condition if the lowest end of the support of the buyer’s valuation is greater than $k$ times

\[ C \]

that of the sellers’ valuation. We show that TL is a necessary and sufficient condition for almost surely positive VCG surplus in the limit in the model where new sellers are added consecutively on a line.

Priors satisfy the TLS1 condition if the lowest end of the support of the buyer’s valuation is greater than the sum of the two endpoints of the support of sellers’ valuation. Consider a sequence of graphs constructed by connecting new nodes to the hub of a star graph. We show that TLS1 is a necessary and sufficient condition for almost surely positive VCG surplus in the limit in this case. Note that TLS1 is a stronger condition than TL when $k = 2$.

We then generalize these conditions to sequences of graphs with special properties. A sequence of graphs satisfies the line inclusion property if given any arbitrary natural number $m$, there is an element in the sequence with a subgraph which is a line graph of size $m$. We show that TL is a necessary and sufficient condition for almost surely positive VCG surplus in the limit for such a sequence.

A sequence of graphs satisfies the $C$-preservation property if (a) each graph in the sequence has $C$ critical sellers and (b) new nodes can be connected to only nodes representing critical sellers. We show that a generalization of TLS1 is required to obtain almost surely positive VCG surplus in the limit: the lowest end of the support of the buyer’s valuation must be greater than the sum of $C$ times the sellers’ highest valuation and $k - C$ times the sellers’ lowest valuation.

In the next section, we briefly review the relevant literature. The following section presents the model, definitions and a brief note on VCG mechanisms. The next two sections contain the results and discussion, respectively. All proofs are presented in the Appendix. We conclude with a few remarks.

2. Literature

In a classic paper, Myerson and Satterthwaite (1983) showed that ex-post efficiency cannot be attained in a model of bilateral trade with two-sided asymmetric information. But there are other models with asymmetric information where this is not true, e.g., partnership dissolution (Cramton et al., 1987), transfer of one indivisible item from one seller to two prospective buyers (Makowski and Mezzetti, 1993) or bargaining among many buyers with unitary demand and many sellers with unitary endowments (Williams, 1999). The efficient mechanisms derived in these papers, however, require information about the underlying priors—they are, therefore, subject to Wilson’s Critique discussed above.

Post Myerson and Satterthwaite (1983), many authors have investigated asymptotic efficiency of market mechanisms extensively (Satterthwaite and Williams, 1989b, a; Gresik and Satterthwaite, 1988; McAfee, 1992; Rustichini et al., 1994; Cripps and Swinkels, 2006; Fudenberg et al., 2007; Williams, 1991; Satterthwaite and Williams, 2002). In such models, an increase in the number of agents implies that the incentive of misreporting when others are reporting truthfully becomes smaller. Consequently, incentive compatible mechanisms begin to approximate the first best. The validity of this reasoning depends on the setting. For instance, this is not true in problems involving public goods (e.g., Mailath and Postlewaite (1990)a). See the review article by Jackson (2000) for elucidation.

Asymptotic properties of VCG mechanisms have been examined in various contexts. Tideman and Tullock (1976) investigated the collective choice problem where agents are required to pay a “Clarke Tax” to get alternatives according to their preference. They conjectured that the per capita budget surplus converges to zero as number of individuals becomes large. Green et al. (1976) demonstrated the same property in an economy where individuals have to make a collective decision on a public project. Rob (1982) extended the result to choice between two public projects and

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3 See Bergemann and Valimäki (2006) for a survey.
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