Finite Time Robust Feedback Nash Equilibrium for Linear Quadratic Games

Nain de la Cruz * Manuel Jimenez-Lizarraga *
* Department of Physical and Mathematical Sciences, Autonomous University of Nuevo Leon, San Nicolas, Nuevo Leon, Mexico.

Email: nca200881@gmail.com, manalejimenez@yahoo.com

Abstract: In this paper we consider the solution for an N players non-cooperative differential game affected by some sort of uncertainties. The problem analyzed is linear quadratic in nature, and the uncertainty affecting the game is square integrable, which is seen as a malicious fictitious player trying to maximize the cost function of each player. In order to find the solution to this problem we solve a robust form of the Hamilton-Jacobi-Bellman equation, which allows us to find the robust equilibrium strategies for each player and in turn to solve a Coupled Riccati Differential Equation.

© 2017, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: LQ Games, Robust Control, Coupled Riccati Equations.

1. INTRODUCTION

Differential games stand as a suitable framework to model strategic interaction between different agents (known as players), where each of them is looking for the minimization or equivalently the maximization of his own individual criterion [Engwerda, 2005], [Basar and Olsder, 1999]. In such a multi-player scenario, none of the players are allowed to maximize their profits or objectives on the expense of the rest of the players. Therefore, the solution of the game is given in a form of "equilibrium of forces".

Among different types of solutions the so-called Nash equilibrium is the most extensively used in the game theory literature. In this solution none of the players can improve their criteria by unilaterally deviating from their Nash strategy. When the full state information is available to all the players to realize their decision strategy in each point of time, this is called a feedback Nash equilibrium [Engwerda, 2005], [Basar and Olsder, 1999], [Friedman, 1971]. In order to find such feedback strategies the optimal control tools are applied, specifically an equivalent form of the Hamilton-Jacobi-Bellman equation is required to solve for each of the players. In the case of the non-cooperative Nash equilibrium solution framework, each player deals with a single criterion optimization problem (the standard optimal control problem), with the actions of the remainder players taking the fixed equilibrium values.

Although the notion of robustness is such an important feature in the control theory there are not many studies of dynamic games that are affected by some sort of uncertainties or disturbances. Some recent developments on this topic can be mentioned. In [Jimenez-Lizarraga and Poznyak, 2007] it is presented a notion of Open Loop Nash Equilibrium (OLNE) where the parameters of the game are within a finite set and the solution is given in terms of the worst-case scenario, that is, the result of the application of certain control input (in terms of the cost function value) is associated with the worst or least favorable value of the unknown parameter. The article of [Jank and Kun, 2002] shows also an OLNE and derive conditions for the existence and uniqueness of such an equilibrium; however, in this case they considered that the uncertainty belongs to a Hilbert functional space and enters adding up into the time derivative of the state variables. The work of [Jungers et al., 2008] deals with a game with polytopic uncertainties that reformulated the problem as a nonconvex coupling between Semi-Definite Programming to find the Nash type controls. Other related approaches include: using the Nash strategy to design robust controls for linear systems [Chen and Zhou, 2001]. One of the ways to deal with uncertainties is to view them as an exogenous input (a fictitious player) [Chen et al., 1997]. In [van den Broek et al., 2003], the definition of equilibria is extended to deal with two cases: a soft-constrained formulation, based on [Jank and Kun, 2002], where the fictitious player is introduced in the criteria via a weighting matrix.

In this work, inspired in the work of [Jank and Kun, 2002], [Engwerda, 2005] and [Engwerda, 2014] we analyze a deterministic N-player LQ Differential Game case, considering finite time horizon in the performance index and an $L^2$ perturbation that affects the game. The feedback Nash strategies are given for the noncooperative N players non-zero-sum game, that to the best of our knowledge have not been treated, in the solution of a set of robust version of the HJB equations where the perturbation is considered as a fictitious player trying to maximize the cost of each $i$-th player, and the rest of the players compute their Nash strategies considering that maximization from their own point of view. To summarize, the contributions of this work are as follows:

- To introduce a robust form of the HJB equation for N players non-cooperative LQ games.
- Based on such a formulation give the solution for the finite as well as the infinite time horizon of the LQ uncertain game.
- Simulation comparisons of the robust and non-robust Feedback Nash controls.

The development of this paper is as follows first, in section 2, we state, formally, the nature of the problem which we are dealing with. We define the dynamics of the problem analyzed...
and the type of functional cost we have to minimize. After that in section 2.2 we define a theorem, based in dynamic programming, that is going to be the basis for this development. Finally, section 3 follows with a numerical example. The purpose of this last section is to show how to apply the formulas obtained in section 2 and then compare our results against a finite time differential game which does not consider perturbation in the solution of the problem, which is the common problem treated, but the system itself is affected by some sort of perturbation. Also, in section 3, we compare our results against an infinite time differential game.

2. PROBLEM STATEMENT

Let us consider the LQDGF affected by some sort of uncertainty where the players' dynamics are covered by the linear ordinary differential equations (ODEs):

$$\dot{x}(t) = A(t)x(t) + \sum_{j=1}^{N} B_j(t) u_j(t) + E(t)w(t)$$

$$x \in \mathbb{R}^n; x(0) = x_0; u_j \in \mathbb{R}^{m_j}, t \in [0,T]; T < \infty$$

where $j$ represents the number of the player, $A(t) \in \mathbb{R}^{n \times n}$, and $B_j(t) \in \mathbb{R}^{n \times m_j}$ are the control matrices, $x \in \mathbb{R}^n$ is the state vector of the game and $u_j \in \mathbb{R}^{m_j}$ is the control strategy for each $j$-player. $w(t)$ is a finite disturbance in the sense that $\int_0^T w^2(t)dt$ converges, that is $w(t)$ is square integrable or stated in another way $w(t) \in L^2_2$ vector in a square-integrable space function entering the system through the matrix $E(t) \in \mathbb{R}^{n \times q}$. For this $N$-players model consider the following individual aim performance:

$$J_i(t, x, u_i, w) = \frac{1}{2} \left[ \int_0^T \left( x^T(t) Q_i(t) x(t) + \sum_{j=1}^{N} u_j^T(t) R_{ij}(t) u_j(t) - w^T(t) W_i(t) w(t) \right) dt + x^T(T) Q_{iT} x(T) \right],$$

$$i = 1, 2, \ldots, N$$

The performance index for each $i$-player is given in standard Bolza form, the strategy for the player $i$ is $u_i$ while $u_i$ are the strategies of the rest of the players (i is the counter-coalition collection of players interacting with the $i$-th player). The term $w^T(t) W_i(t) w(t)$ is the unknown uncertainty, which is trying to maximize the cost $J_i$ from the point of view of the $i$-th player. The cost matrices are assumed to satisfy: $Q_i(t) = Q_{iT}(t) \geq 0$, $Q_{iT} = Q_{iT}^T \geq 0$ and $W_i(t) = W_i^T(t) > 0$, (symmetric and semipositive/positive definite matrices) $R_{ij}(t) = R_{ij}^T(t) > 0$ and $R_{ij}(t) = R_{ij}^T(t) > 0$. Assume also that the players have access to the full state information pattern, that is, they measure $x(t), \forall t \in [0,T]$. Remark 1: The time-dependence notation is intentionally omitted henceforth since it is clear from the previous context.

2.1 Memoryless Robust Feedback Nash Equilibrium

Next we introduce the worst-case uncertainty from the point of view of the $i$-th player of the $j$-th according to the complete set of controls $u^j$: $(j = 1, \ldots, N)$ [Jank and Kun, 2002];

$$F_i(t, x, u_i, u_i^0, w^0(u_i, u_i)) := \sup_{w \in L^2_2} J_i(t, x, u_i, u_i, w(u_i, u_i))$$

In this paper we want to extend the Robust Nash Equilibrium notion previously introduced in [Jank and Kun, 2002], [Engwerda, 2005], for an open loop information structure to a memoryless full state information and to $N$ players case. Definition 1. The control strategies are said to be in a Robust Feedback Nash Equilibrium if:

- for any admissible strategy

$$(u_i, u_i) \in U_{adm}, (i = 1, \ldots, N)$$

and the corresponding maximizing uncertainty $w^0(u_i, u^0_i)$ the next set of inequalities hold:

$$F_i(t, x, u_i^0, u_i^0, w^0(u^0_i, u^0_i)) \leq F_i(t, x, u_i, u_i, w(u_i, u_i))$$

(3)

2.2 Robust Dynamic Programming for LQ Games

Consider the $N$-tuples of continuous strategies $(u_i, u_i)$ and the robust value function from the point of view of the $i$-th player as:

$$V_i(t, x) = \min_{u_i \in U_i} F_i(t, x, u_i, u_i, w^0(u_i, u_i));$$

$$i = 1, 2, \ldots, N.$$ Remark: In previous important works dealing with the design of robust H∞ controllers using a dynamic game approach [Basar and Bernhard, 2008], [Aliyu, 2011], it is already found a robust version of the value function. However, to the best of our knowledge, the case for $N$ players LQ noncooperative games, which is presented here, has not been introduced.

Applying the previous definition and the dynamic programming principle, we arrive to the following result for the $N$ players LQ game:

**Theorem 1.** Assume that $\frac{\partial V_i(t, x)}{\partial x}$, $\frac{\partial V_i(t, x)}{\partial u_i}$ exist and are continuous for all $x, t$. Then $V_i: [0,T] \times \mathbb{R}^n \to \mathbb{R}$ satisfy the following partial differential equation:

$$-\frac{\partial V_i(t, x)}{\partial t} = \min_{u_i \in U_i} \max_{w \in L^2_2} \left\{ \left( \frac{\partial V_i(t, x)}{\partial x} \right)^T \left( Ax + \sum_{j=1}^{N} B_j u_j + Ew \right) + x^T Q_i x + \sum_{j=1}^{N} u_j^T R_{ij} u_j - w^T W_i w \right\},$$

(4)

$$V_i(T, x) = x^T(T) Q_{iT} x(T)$$

where $x(t)$ which is the state variable solution to the ordinary differential equation (1). If the assumptions mentioned above are satisfied then $u_i$ provides a Robust Feedback Nash Equilibrium, where:

$$u_i = \arg \min_{u_i \in U_i} \left\{ \left( \frac{\partial V_i(t, x)}{\partial x} \right)^T \left( Ax + \sum_{j=1}^{N} B_j u_j + Ew \right) + x^T Q_i x + \sum_{j=1}^{N} u_j^T R_{ij} u_j - w^T W_i w \right\},$$

(5)

and

$$w = \arg \max_{w \in L^2_2} \left\{ \left( \frac{\partial V_i(t, x)}{\partial x} \right)^T \left( Ax + \sum_{j=1}^{N} B_j u_j + Ew \right) + x^T Q_i x + \sum_{j=1}^{N} u_j^T R_{ij} u_j - w^T W_i w \right\},$$

(6)
دریافت فوری
متن کامل مقاله
امکان دانلود نسخه تمام متن مقالات انگلیسی
امکان دانلود نسخه ترجمه شده مقالات
پذیرش سفارش ترجمه تخصصی
امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
امکان دانلود رایگان ۲ صفحه اول هر مقاله
امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
دانلود فوری مقاله پس از پرداخت آنلاین
پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات