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Applications of stochastic optimal control/dynamic programming to international finance and debt crises

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Abstract

In July 1997, the economies of East Asia became embroiled in one of the worst financial crises of the postwar period. In 2001 Argentina defaulted on its sovereign debt. Prior to the crises, the markets and the International Monetary Fund viewed these economies as models of growth and stability. We use stochastic optimal control and dynamic programming to model an optimal foreign debt and show why divergences of the actual debt from the optimal make the economies vulnerable to crises. These divergences imply measurable warning signals. We provide examples of the derived warning signals for Korea and Argentina.

Keywords: Optimal debt; Dynamic programming; Stochastic optimal control; Korea; Argentina; Warning signals of debt crises

0. Introduction

A major part of the mathematical finance literature has been concerned with intertemporal optimization and risk management of portfolios. The seminal work was by Robert Merton, *Continuous Time Finance* (1990). Wendell Fleming, his co-authors and others influenced by his work, have made the subsequent major contributions. The articles use the techniques of stochastic optimal control/dynamic programming (SOC/DP), and successively consider ever more interesting, difficult and realistic modeling of both the uncertainty and the

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constraints. Comprehensive evaluations of this mathematical literature are by Fleming [1–3]. Recently, Fleming and Stein [5,6], Stein [7], Stein and Lim [8], Stein and Paladino [9] have applied the techniques of SOC/DP to the modeling of debt crises. The aim of this interdisciplinary paper is to explain the logic of the modeling and the ability of this approach to derive warning signals of debt crises.

In July 1997, the economies of East Asia became embroiled in one of the worst financial crises of the postwar period. Yet, prior to the crisis, these economies were seen as models of economic growth experiencing sustained growth rates that exceeded those earlier thought unattainable. Similarly in 1998, the financial markets and the International Monetary Fund viewed Argentina as a model of stability and growth. However in 2001–02 the Argentine economy defaulted on its huge debt. Why did the market not anticipate the crises?

To this end, we use SOC/DP to model an optimal foreign debt, and show why "divergences" lead to debt crises. The important point here is that these models suggest important variables which may serve as warning signals to predict crises. We provide examples of these warning signals for Korea and Argentina.

1. The logic of a debt crisis: prototype model

A debt crisis is likely to occur when the actual external debt is excessive or "unsustainable". In order to assess whether the debt is "unsustainable" we need a concept of an optimal debt as a benchmark. Excess debt can then be measured as the deviation of the actual debt from the optimal debt.

It is realistic to assume that the country will have a debt crisis if the attempt to service the debt requires a decline in consumption below a tolerable level, or requires a drastic decline in consumption. To see this first, consider Eq. (1) which describes the change in the debt $\mathrm{d}L_t$, where L is the real external debt.

$$\mathrm{d}L_t = (I_t - S_t)\,\mathrm{d}t = (C_t + I_t + r_t L_t - Y_t)\,\mathrm{d}t,\ S_t = Y_t - r_t L_t - C_t. \tag{1}$$

Debt rises because consumption C_t plus investment–capital formation I_t plus the debt service $r_t L_t$ exceeds Y_t the GDP. Alternatively, the change in the debt is $(I_t - S_t)\,\mathrm{d}t$ investment less saving S_t over the period. In the Latin American countries the debt has risen due to high consumption/low social saving by the public plus the private sectors. In the Asian countries, the high investment has produced the external debt. For example, there were speculative bubbles in asset prices that raised the anticipated returns and investment. The rise in investment led to a capital inflow and an increase in the external debt.

The external debt has to be serviced and that would clearly affect consumption. We can see this by writing consumption at some time $s = t + \Delta t$, equation

$$C_s\,\mathrm{d}t = (Y_s - r_s L_s - I_s)\,\mathrm{d}t + \mathrm{d}L_t. \tag{2}$$

Consumption is equal to the GNP, which is equal to the GDP less the debt service $(Y_s - r_s L_s)$, less investment I_t plus new borrowing $\mathrm{d}L_t$.

Focus now on the behavior of the two stochastic variables—real GDP and real interest rate. If bad shocks reduce the GDP and raise real interest rates, and investment falls to a minimum level $I_s = I_{\min}$ then consumption may have to be reduced—unless there is new