Partial attribute reduction approaches to relation systems and their applications

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1. Introduction

Attribute reduction is the process whereby dispensable attributes are removed from a given database of knowledge while maintaining consistency. It is among the most important topics in rough set theory [1,2]. Pawlak was the first to propose the concept of attribute reduction for decision tables. Pawlak and Skowron [3,4], Skowron and Rauszer [5] proposed an algorithm for attribute reduction based on a discernibility matrix with equivalence relations. Skowron and Rauszer [6] were the first to propose the concept of the discernibility matrix. As this matrix is intuitive and easy to understand, attribute reduction based on it is efficient. Many authors [7–15] have proposed similar algorithms with an extended discernibility matrix for different types of attribute reduction. For example, Zhang et al. [16] studied distributive reduction with an appropriate discernibility matrix. Liu et al. [17] developed a unified algorithm based on invariant matrices for three types of reduction in decision tables.

An equivalence relation is too restrictive for many applications; therefore, several authors [18–20] have recently studied certain types of reduction using dominance relation-based attribute reduction. For a given decision table, Wei et al. [21] derived a compacted decision table that can preserve all information contained in the original. Yamany et al. [22] proposed an intelligent optimization method called the “flower search algorithm,” which adaptively searches for optimal attributes, for the fitness function used in rough sets-based classification. Liu et al. [23] proposed a general attribute reduction algorithm for relation decision systems. Zhang et al. [24], Zhang and Xu [25,26] extended the concepts of the lower and upper approximation reduction to ordered information systems. In 2012, Xu et al. [27] investigated upper approximation reduction in an ordered information system with fuzzy decision making. Shao et al. [28] investigated granular reduction in formal fuzzy contexts. Sun et al. [29] recently studied multi-criteria group decision making based on multi-granulation fuzzy rough sets over two universes. Ju et al. [30] considered the design of cost-sensitive rough set models using a multi-granulation strategy.

A number of researchers [31–36] have developed approximation reduction methods based on general binary relations. This paper considers this type of reduction in general relation systems. We first define the concept of lower approximation reduction and its dual reduction for relation systems. An lower approximation reduction is a partial reduction. As an application of such a reduction, we derive lower and upper approximation reduction algorithms. As a special case, we establish the relationship between positive region reduction and lower approximation reduction in a decision table.

The remainder of the paper is organized as follows: In Section 2, we recall some basic notions and notations related to relations and relation systems. In Section 3, we propose the concept of, and provide an algorithm for, lower approximation reduction. As the dual of lower approximation reduction, Section 4 consid-
ers X-upper approximation reduction. Sections 5 and 6 detail lower and upper approximation reduction for relation decision systems, respectively. Section 7 shows that the positive region reduction for decision tables is a special case of lower approximation reduction and Section 8 provides two examples to verify our theoretical results. Section 9 contains the conclusions of this study.

2. Preliminaries

In this section, we rehearse some basic definitions and properties of binary relations and relation decision systems. Let $U = \{x_1, x_2, \ldots, x_n\}$ be a finite set of objects called the universe set. Suppose that $R$ is an arbitrary relation on $U$. Recall that the left and right $R$-relative sets of an element $x$ in $U$ are defined as $l_R(x) = \{y \in U : yRx\}$ and $r_R(x) = \{y \in U : yRx\}$, respectively. Based on the right $R$-relative set, the lower and upper approximations of $X \subseteq U$ are defined as [15]

$$R(X) = \{x \in U : r_R(x) \subseteq X\} \text{ and } \overline{R}(X) = \{x \in U : r_R(x) \cap X \neq \emptyset\},$$

respectively. Wang et al. [35] proposed the concept of relation decision systems, and we generalize this concept such that the decision attribute no longer needs to be an equivalence relation [37].

**Definition 2.1** [35,37]. Let $U = \{x_1, x_2, \ldots, x_n\}$ be a finite universe set and $A$ be a family of binary relations on $U$. Then, $(U, A)$ is called a relation system. In addition, if $A = U \cup D$ and $C \cap D = \emptyset$, $(U, U \cup D)$ is called a relation decision system, and $C$ is called the condition attribute set and $D$ the decision attribute set. If $R_C = \bigcap_{C \in R} R \subseteq \bigcup_{D \in D} (U, C \cup D)$ is said to be consistent; otherwise, $(U, U \cup D)$ is said to be inconsistent.

The following proposition is elementary, and hence we omit a proof:

**Proposition 2.1.** Let $(U, A)$ be a relation system and $X, Y \subseteq U$.

1. If $B \subseteq A$, then $R_B \subseteq R_B$ and $R_B(Y) \subseteq R_B(Y)$.
2. If $X \subseteq Y$, then $R_B(X) \subseteq R_B(Y)$.
3. $R_B(X) \cup R_B(Y) \subseteq R_B(X \cup Y)$.

3. The X-lower approximation reduction for relation systems

Let $(U, A)$ be a relation system. For any given subset $X \subseteq U$, we consider a reduction type that keeps the lower approximation $R_B(X)$ unchanged. We now provide its definition.

**Definition 3.1.** Let $(U, A)$ be a relation system. For a given subset $X \subseteq U$ and $\emptyset \neq B \subseteq A$, $B$ is called the $X$-lower approximation reduction of $(U, A)$ if $B$ satisfies the following conditions:

1. $R_B(X) = R_B(X)$.
2. If $B' \subseteq B$, $R_B(X) \neq R_{B'}(X)$.

Since $R_B(U) = U$, each singleton set $\{r\} \cap A$ is a $U$-lower approximation reduction of $(U, A)$. Thus, we assume that $X \neq U$ throughout this paper.

Let $(U, A)$ be a relation system, where $U = \{x_1, x_2, \ldots, x_n\}$ and $A = \{R_1, R_2, \ldots, R_k\}$. For subset $X \subseteq U$, we define the discernibility matrix $M = (m_{ij})_{k \times s}$ as follows:

$$m_{ij} = \begin{cases} \{r \mid (x_i, x_j) \notin R_r\}, & \text{if } x_i \in R_B(X) \text{ and } x_j \notin X \text{ and } s \neq 0, \\ \emptyset, & \text{otherwise} \end{cases},$$

where $s = |R_B(X)|$ and $t = |X'|$ are the cardinalities of sets $R_B(X)$ and $X' = U - X$, respectively. The computational complexity of the discernibility matrix $M = (m_{ij})_{k \times s}$ is $O(st)$.

Now we provide the $X$-lower approximation reduction algorithm. We use the following lemmas:

**Lemma 3.1.** Let $(U, A)$ be a relation system and $X \subseteq U$. If $x_i \in R_B(X)$ and $x_j \notin X$, $m_{ij} \neq \emptyset$.

**Proof.** Suppose that $x_i \in R_B(X)$, $x_j \notin X$ if $m_{ij} = \emptyset$; then, $(x_i, x_j) \in R_B$ for each $R_B \in A$. Thus, $(x_i, x_j) \in R_B$ and $x_j \in r_B(x_i)$. Since $x_i \in R_B(X)$, $x_j \in r_B(x_i)$. This is contradictory with the assumption. \square

The $X$-lower approximation reduction algorithm is based on the following theorem:

**Theorem 3.1.** Let $(U, A)$ be a relation system, and $X \subseteq U$. Then, the following conditions are equivalent:

1. $R_B(X) = R_B(X)$.
2. If $m_{ij} \neq \emptyset$, $B \cap m_{ij} \neq \emptyset$.

**Proof.** Suppose that $x_i \in R_B(X)$, $x_j \notin X$ if $m_{ij} = \emptyset$; then, $(x_i, x_j) \in R_B$ for each $R_B \in A$. Thus, $(x_i, x_j) \in R_B$ and $x_j \in r_B(x_i)$. Since $x_i \in R_B(X)$, $x_j \in r_B(x_i)$. This is contradictory with the assumption. \square

According to Corollary 3.1, for any given subset $X \subseteq U$, we can give an $X$-lower approximation reduction algorithm for relation system $(U, A)$ as follows:

1. Find the discernibility matrix $M = (m_{ij})_{k \times s}$, where $s = |R_B(X)|$ and $t = |X'|$.
2. Transform the discernibility function $f$ from its CNF $f = \wedge_{m_{ij} \neq \emptyset} (\vee m_{ij})$ into a DNF $[38] f = \vee_{i=1}^{s} (\vee_{B \in A} R_B)$. (But $A \subseteq A$).
3. Red($A$) = $\{B_1, B_2, \ldots, B_s\}$ and Core($A$) = $\cap_{B \in A} B$. End the algorithm.

According to Corollary 3.1, for any given subset $X \subseteq U$, we can give an $X$-lower approximation reduction algorithm for relation system $(U, A)$ as follows:

**Example 3.1.** Let $U = \{1, 2, 3, 4\}$ and $A = \{R_1, R_2, R_3, R_4\}$. Relations $R_1$, $R_2$, $R_3$, and $R_4$ on $U$ are respectively defined by the following relational matrices:

$$M_{R_1} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \quad M_{R_2} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \quad M_{R_3} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \quad M_{R_4} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}.$$
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