Maximum nondiffracting propagation distance of aperture-truncated Airy beams

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ABSTRACT

Airy beams have called attention of many researchers due to their non-diffracting, self-healing and transverse accelerating properties. A key issue in research of Airy beams and its applications is how to evaluate their nondiffracting propagation distance. In this paper, the critical transverse extent of physically realizable Airy beams is analyzed under the local spatial frequency methodology. The maximum nondiffracting propagation distance of aperture-truncated Airy beams is formulated and analyzed based on their local spatial frequency. The validity of the formula is verified by comparing the maximum nondiffracting propagation distance of an aperture-truncated ideal Airy beam, aperture-truncated exponentially decaying Airy beam and exponentially decaying Airy beam. Results show that the formula can be used to evaluate accurately the maximum nondiffracting propagation distance of an aperture-truncated ideal Airy beam. Therefore, it can guide us to select appropriate parameters to generate Airy beams with long nondiffracting propagation distance that have potential application in the fields of laser weapons or optical communications.

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1. Introduction

During the last decade, Airy beams have called attention of many researchers due to their non-diffracting self-healing and transverse accelerating features. They may have many potential applications such as in optical micromanipulation [1,2], spatiotemporal light bullets [3,4], plasma physics [5], optical communication [6–9] etc.

Like other ideal nondiffracting waves, the ideal Airy beam can propagate over infinite distance resisting the diffraction effects, but it contains an infinite power flux [10]. In addition, by its very nature, an ideal Airy beam is “weakly confined” since its oscillating tail decays very slowly [11]. Therefore, it is not physically realizable. To overcome this problem, Siviloglou obtained a finite energy Airy beam solution by means of an ideal Airy beam modulated by an exponentially decaying function at \( z = 0 \) in 2007 [11,12]. However, in practice, a finite energy Airy beam is normally spatially truncated by an aperture on the ideal Airy beam. And the spatial truncation is the most effective and realistic option. If the geometrical size of the limiting aperture greatly exceeds the spatial features of the ideal Airy beam, the diffraction process is considerably “slowed down” over the intended propagation distance and for all practical purposes these beams are called “diffraction-free” [12]. Nonetheless, an interesting problem that is hardly discussed in all the published literature on finite energy Airy beams is over what a nondiffracting distance the finite energy Airy beam can propagate in free-space. To our knowledge, the nondiffracting distance of a finite energy Airy beam was evaluated by comparing the corresponding intensity profile of the Airy beam propagating to different range with the one at the initial plane. If the spatial FWHM width of the main lobe remains almost invariant up to a certain distance, the maximum nondiffracting propagation distance was achieved. No analytical formula has been given except one (Eq. (31)) given in Ref. [13]. But Eq. (31) in Ref. [13] would lead to a wrong result that will be demonstrated in this paper.

The aim of this paper is to provide a better understanding of the fundamental nature of the local spatial frequency methodology applied to Airy beams. In Section 2 we derive a formula that would provide some insight regarding the critical transverse extent of physically realizable Airy beams after which their oscillating tail must dampen. Section 3 we develop an explicit expression to evaluate the maximum nondiffracting propagation distance of an aperture-truncated ideal Airy beam. In Section 4, we use the model of Airy beams generated by finite apertures to demonstrate the validity of the expressions. Finally, a brief conclusion is given.

2. The critical transverse extent of physically realizable Airy beams

Airy beam is a solution of the paraxial wave equation. The Sommerfeld radiation condition tell us that a wave equation cannot have waves
coming from an infinite distance [13]. It means that the incoming wave must be generated at a finite distance, implying that finite energy Airy beams must have finite transverse extent and thus a finite propagation distance.

We begin our analysis by considering the complex amplitude of

\[ U(s, \zeta) = Ai(s) \exp(as) \tag{1} \]

Here, \(U(s, \zeta)\) is the complex amplitude depending on the normalized coordinates \(s = x/x_0\) and \(\zeta = z/kx_0^2\), with \(x_0\) being an arbitrary transverse scaling factor and \(k = 2\pi/\lambda\) being the wave number. \(Ai(\cdot)\) is the Airy function and \(a\) is the exponential decay factor.

Fig. 1 shows the normalized transverse profile of an exponentially decaying Airy beam. We can see that the complex amplitude of an ideal Airy beam modulated by an exponentially decaying function oscillates with respect to the transverse coordinate. For an oscillating function, a local spatial frequency function can be used to describe its spatial oscillating features. It can be defined as

\[ f(x) = \frac{1}{2\pi} \frac{d\phi(x)}{dx} \tag{2} \]

where \(\phi(x)\) is the wave front of an oscillating function.

By using the zero-point coordinates of the complex amplitude of an exponentially decaying Airy beam, an analytical formula describing the local spatial frequency of the Airy beam can be calculated through nonlinear curve fit [14].

\[ f(x) = \frac{1}{2\pi x_0^{\frac{3}{2}}} \sqrt{x} \tag{3} \]

Ref. [13] states that: For an Airy beam, the distance between two consecutive peaks in the transverse intensity profile should not become smaller than the wavelength of the light used. It means that one must have a critical transverse extent for a physically realizable Airy beam. The oscillating tail extending over this critical value must dampen. Because the local spatial frequency of a given spatial point is equal to the reciprocal of the oscillating period near the point, the local spatial frequency of an Airy beam near the critical point is \(f(x) = 1/\lambda\). The critical transverse extent of physically realizable Airy beams can then be expressed as

\[ |x|_{\text{max}} = \sqrt{\frac{k^2}{x_0^3}} = 4x_0^{3/2} \tag{4} \]

Eq. (4) tells us that physically realizable Airy beams can only have rays coming from region \(|x| \leq |x|_{\text{max}}\). And the critical transverse extent of physically realizable Airy beams is dependent on the transverse scaling factor \(x_0\) and light wavelength \(\lambda\), as shown in Fig. 2. We can see that the critical transverse extent increases with increase of \(x_0\) or decrease of \(\lambda\). The Sommerfeld radiation condition imposes a restriction on Airy beam. They must be confined in a region of finite extent at \(z = 0\). And they can only propagate without diffraction within a finite region of space.

In Section 2, we gave the critical transverse extent \(|x|_{\text{max}}\) on the plane \(z = 0\) in order for physically realizable Airy beams. For example, \(|x|_{\text{max}}\) would reach 98.5 m if \(\lambda = 632.8\) nm and \(x_0 = 100\) μm. This critical extent is too large to be experimentally realizable because the aperture size of a general physical device (such as a spatial light modulator) is much smaller than this value. The transverse extent of experimentally generated finite energy Airy beams will surely be truncated by the aperture (because of lack of space and power) at the onset of propagation (\(z = 0\)). The position where the beam is truncated is located at the edge of the aperture. The peak of the main lobe of an aperture-truncated Airy beam can be regarded as a caustic emerging from the rays emerging sideways [15]. Therefore, the maximum nondiffracting propagation distance of an aperture-truncated Airy beam is determined by the position of the truncation point.

The propagation trajectory of the peak of the main lobe of an aperture-truncated Airy beam is described by the theoretical relation

\[ x_d = \frac{\lambda^2}{16\alpha^2x_0^3} z^2 + a_1 \tag{5} \]

where \(x_d\) is the transverse shift of the peak of the main lobe of the Airy beam with propagation distance \(z\). And \(a_1 = -1.0187297\ldots\) is the position of the peak of main lobe at the onset of propagation.

As showed in Fig. 3, the coordinate of the collinear point between the local wave vector of an Airy beam and the tangent of its acceleration trajectory under the paraxial condition should satisfy [14].

\[ \frac{dx_d}{dz} = \frac{x_d + z}{z} \tag{6} \]

Substituting Eq. (5) into Eq. (6), we can derive an analytical formula to describe the maximum nondiffracting propagation distance of an aperture-truncated Airy beam; specifically,

\[ z_{\text{max}} = 2kx_0^{3/2} \sqrt{|x_{\text{cut}}| - |a_1| x_0} \tag{7} \]
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