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The power of dimensional analysis in production systems design

Giovanni Miragliotta *

Politecnico di Milano Dept of Management, Economics and Industrial Engineering, Via Lambruschini 4/B, 20156 Milano, Italy

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ABSTRACT

Dimensional Analysis (DA) is a well-known methodology in physics, chemistry and other traditional engineering areas. In its simplest form, DA is used to check the meaningfulness of a set of equations (dimensional homogeneity). In the last century, the dimensional theory has been profoundly investigated: its highest achievement is the Buckingham theorem (or pi-theorem), which states that any equation modelling a physical problem can be rearranged in terms of dimensionless ratios, thus saving variables to be handled, and especially enriching the inner physical knowledge of the studied phenomenon.

In this paper we investigate how DA can be applied to Operations Management (OM) topics and which benefits it can bring to researchers in this area. A literature review is performed to clarify the main operative issues regarding DA application (assumptions and limitations); then existing applications of DA to OM are explored, pointing out that few researchers have tried to apply this methodology in the OM research field.

Stemming from this analysis, we applied the pi-theorem to the design of a Flexible Manufacturing System. A complex problem, requiring 13 dimensional quantities to be expressed, is first studied via simulation; then DA is applied, reducing the number of variables to 9 dimensionless ratios. The reduced problem has a suitable size to be analytically explored and a regression model is formulated which, compared with the simulation study, offers the same precision in analysing the FMS behaviour, being more compact and powerful. This application shows the potential of DA in OM research, and will hopefully draw the attention of researches to this powerful, but unfamiliar and therefore neglected, methodology.

1. Introduction

Dimensional Analysis (DA) is a well-known methodology used in physics and traditional engineering areas in order to empower the model formulation and to cut efforts in the empirical assessment phases. Its roots can be traced back to the 19th century with the first attempts to analyse the dimensional structure of a physical law, but only in recent years, thanks to the outstanding work of Taylor (1974), all the elements concerning the theoretical foundation of this methodology have been perfected.

The highest achievement of DA is the Buckingham theorem (or pi-theorem or Π-theorem), which states that any equation modelling a physical problem can be rearranged and simplified using a set of dimensionless variables (or numbers, or ratios) so that the number of variables originally used to describe the problem can be reduced by the number of independent fundamental physical quantities used in the original equation. In this way, the modeller can save variables to handle and, above all, get a richer knowledge of the studied phenomenon.

Despite its patent usefulness, several difficulties in understanding the theorem's foundations prevent a more widespread adoption in some research communities: "which assumptions should be satisfied to apply the theorem?", "what is its extent of validity?", "how do we concurrently apply this methodology with other methodologies such as regression?" are the typical questions asked by those researchers unfamiliar with the method. Hence the Buckingham Theorem (BT) is very often used in traditional engineering and physics subjects, where the DA was historically conceived and refined, while it is uncommon to find relevant applications in other research fields, such as Operations Management (OM) and Economics just to name a few.

The purpose of this paper is to provide a brief but effective introduction to the BT, so that more researchers could get familiar with its functioning, appreciate its enormous capabilities in supporting model development and empirical analysis and eventually include the BT into their "researcher's toolbox". In this regard, the paper is arranged as follows. Section 2 will present a review of the literature concerning the DA, from its introduction to the present day, so as to assess what is the state-of-the-art in the BT's understanding and applications. Section 3 will describe briefly the theorem's assumptions and operative usage, providing a few examples of how it can be applied, so that the reader could

* Tel.:+39 02 2399 2785.
E-mail address: giovanni.miragliotta@polimi.it

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become more familiar with it. Section 4 will then present an application of the BT to the design of a Flexible Manufacturing System (FMS), discussing how quantities and variables should be reinterpreted to properly apply the theorem. Then, Section 5 will put forward some concluding remarks, and provide some suggestions about other OM research areas in which the theorem could be successfully applied.

2. Literature review

Broadly speaking, the Dimensional Analysis (DA) can be defined as a research method to deduce more information about a certain phenomenon relying on the postulate that any phenomenon can be described through a dimensionally homogeneous equation. We will clarify some of these introductory elements in Section 3; as for now, it is patent that the dimensional homogeneity does provide an additional information, which can be used to address more effectively the model formulation and validation.

The first known contribution on the topic of DA can be traced back to the heat transfer studies of Fourier (1822). He was the first to conceive of a physical quantity as having two characteristics: a numerical measure and a concept. Almost half a century later, Maxwell (1871) was the first to define and use symbols like $[F]$, $[M]$, $[L]$, $[T]$, $[\theta]$ to identify, respectively, force, mass, length, time and temperature; the great Scottish physicist was also the first to use products of powers of these symbols, calling them "dimensions"; for his outstanding contribution in this field, he can be fairly considered as the father of DA.

The first original application of DA and the first explicit use of dimensionless variables can be ascribed to Rayleigh (1899). Even though his method lacked awareness and is characterised by some arbitrariness, it remains a masterpiece because it represented the first attempt to figure out a more operative usage of the information embedded into an equation's dimensions. His work was then followed by that of Buckingham (1914), whose importance relies on a twofold aspect: not only he formulated the theorem that has his name and that represents the highest expression of DA, but above all he raised the dignity of DA from an empirical technique (already widespread at that time) to a true research methodology endowed with a sound theoretical background. Even though his work was not complete, and required several additions in the following decades (a convincing proof and a full identification of the assumptions, just to name few), he raised the discussion from a laboratory technique to a scientific method, thus drawing the attention of the scientific community on the theme. A subsequent important contribution was made by Bridgman (1922): he concentrated on the applicability of the method, highlighting the theorem's limitation to linear quantities only and deepening the matter of dimensional constants and that of "independent fundamental quantities": his work was thus a theoretical refinement of Buckingham's work.

Thanks to these two authors, DA became more and more known in the scientific community (especially physicists), and new contributions were subsequently presented: Van Driest (1946) was the first to develop a rule for the definition of the dimensionless ratios, while Langhaar (1951) gave a huge refinement to the methodology by providing a new and more accurate proof of the BT, a procedure to assumptions' verification and an automated method to compute the dimensionless ratios, which has remained unchanged to the present day. The work of Langhaar therefore represents a moment of outstanding theoretical and operational consolidation.

In the 1960s we see a profound rethinking of the concept of dimension itself: Quade (1961) deepened the mathematical foundations of the concept of dimension, while Huntley (1967) proposed a further refinement by putting into evidence the independent mechanism through which some dimensions can influence a physical phenomenon: thanks to his contribution, mechanical engineers are now used to treat (in suitable problems) the length and the section of a metal bar (for instance) as having two distinct fundamental quantities (one for the longitudinal length, the other for the transversal width), while physicists are accustomed to distinguishing the mass as a measure of quantity, which is dimensionally distinct from mass as a measure of inertia. Siano (1985) further refined the work of Quade, by introducing a dimensional perspective also for angles and, broadly speaking, orientation dependent problems. Both these works share the same point, which will be discussed in this paper too, i.e. the need to perform a tailored dimensional study depending on the studied problem: the selection of the set of fundamental quantities to be used must leverage on the entire knowledge one has about the phenomenon under study.

In 1974, as we mentioned above, Taylor published a book which can be considered as the bible of DA for engineers: in his monograph, instead of concentrating on the mathematical aspects (already discussed by his predecessors), Taylor moved the attention to the inner nature of the method, highlighting the critical moments of the BT's use and the most important choices to make. Taylor discussed these aspects providing many examples to illustrate the mistakes that can be made in case of unaware or excessively rigid resort to the (already known) procedural steps, for instance in the selection of the fundamental quantities or in the optimal definition of the dimensionless numbers. This book is really complete and well written, but above all Taylor made DA feel alive, not a passive procedure (as ANOVA could be) but an intimate moment of knowledge synthesis and enhancement in the hand of the researcher.

From then on, few significant conceptual developments have made through the years, thanks to the works of Bhargava (1992), dealing with a numerical approach to the DA, of Kokar (2000), who proposed a very useful and effective method to check the correctness and completeness of the proposed dimensionless variables, and of Sonin (2004), who dealt with the case in which some of the starting variables have fixed values in the test bed, thus allowing to further reduce the number of dimensionless variables beyond the statement of Buckingham's theorem itself.

As a consequence, the research efforts moved onto two different streams, one concentrating on the integration of DA with other research methodologies, and the other trying to export the use of DA into new research areas different from traditional physics and engineering ones.

In the first stream we can quote the works of Naddor (1966a) on queuing theory and linear programming, those of Vignaux and Scott (1999) and Vignaux (1991) on regression and data modelling, and finally the work of Mendez and Ordóñez (2005) still on regression. With regard to the second stream we remember the first works of Sasieni et al. (1959), Naddor (1966b) and Flagle et al. (1967), focusing on inventory management, that of de Jong (1967) focusing on economics; more recently Daganzo (1987) applied DA to distribution network design, Willis et al. (1983), Willis and Huston (1990) and Mak and Yeung (1998) worked on a KPIs design procedure, which is weakly inspired by the DA, Vignaux (1986), Vignaux and Jain (1988) and Vignaux (2001) explored the use of BT applications to several small OM problems, Hertkorn and Rudolph (1998, 1999, 2000), and Bruckner and Rudolph (2001) proposed a DA application to case based reasoning and data mining, while Gabbrielli and Braglia (2004) to investment decisions in industrial equipment. Of these works, that of de Jong (1967) is in our opinion very