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Accounting for productivity: Is it OK to assume that the world is Cobb–Douglas? ☆

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ABSTRACT

The development accounting literature almost always assumes a Cobb–Douglas (CD) production function. However, if in reality the elasticity of substitution between capital and labor deviates substantially from 1, the assumption is invalid, potentially casting doubt on the commonly held view that factors of production are relatively unimportant in accounting for differences in labor productivity. We use international data on relative factor shares and capital-output ratios to formulate a number of tests for the validity of the CD assumption. We find that the CD specification performs reasonably well for the purposes of cross-country productivity accounting.

1. Introduction

A critical decision in any development accounting analysis, which aims to decompose GDP per worker into its fundamental components (physical capital, human input and total factor productivity), is the choice of aggregate production function. The standard choice is a Cobb–Douglas (CD) specification, and the common finding is that observed differences in labor productivity cannot be adequately accounted for by differences in physical and human capital. Instead, total factor productivity (TFP) accounts for the lion's share of observed differences in GDP per worker.

Strictly speaking, however, the CD assumption is not an appropriate choice for this type of analysis. Under competitive markets (which is a maintained assumption in development accounting), the CD assumption implies that we should expect zero variation in relative factor shares, when comparing countries at different stages of development. This implication can be resoundly rejected; factor shares do vary from country to country. For example, in a recent data set constructed by Bernanke and Gürkaynak (2001), labor's share falls in a range from 0.53 (Venezuela) to 0.78 (Sri Lanka). Therefore, for the purpose of applied work, a more general CES specification would be a better choice, since it can be consistent with this dimension the cross-country data.

A CES approach, in combination with technology entering in a Harrod neutral way, may lead to new results from development accounting, if one employs an elasticity of substitution (ES) between capital and labor above 1. That is, if the ES is larger than the one implicit in the CD assumption (ES = 1). If the ES > 1 assumption is warranted, the existing accounting

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literature underestimates the importance of rival factors of production. However, if the appropriate choice is an ES below 1, the opposite is true. Hence, the central question is whether an ES above or below 1 is more plausible.[1] The work of Duffy and Papageorgiou (2000) would add weight to the claim that the ES is above 1. Using aggregate cross-country data, they estimate a CES production function and find the ES to be around 1.5.

The first contribution of this paper consists of developing two simple tests which aims to reassess this issue, assuming technology enters the aggregate production function in a Harrod neutral way. The first simple test relates to the predicted correlation between capital's share and the capital-output ratio. If the appropriate choice for ES is above 1, we would expect a positive correlation to emerge, whereas the two variables should be negatively associated in the case where ES is below 1. Using various data sources for capital-output ratios and factor shares we generally find a negative, but insignificant, correlation. This finding is consistent with a production function where the elasticity of substitution is slightly lower than 1.[2] Our second test exploits the information contained in the observed *variation* in factor-shares and capital-output ratios across countries. Using a general CES production function, we show how to relate this observed variation in a simple way to the elasticity of substitution between capital and human input. We find that the elasticity of substitution calibrated in this manner falls in a interval from 0.8 to 0.9. Given this range of estimates for ES, a CD based approach and the more general CES approach (assuming Harrod neutral technological change) will yield very similar results; the role of factors will be only slightly smaller according to the CES-based analysis.

The second contribution of this paper consists of extending the analysis to include a more general form of biased technological change. Caselli (2006) is, to our knowledge, the first to perform development accounting under the assumption that technology manifests itself simultaneously in a Harrod and Solow neutral fashion. Interestingly, under this technological assumption, an ES *below* 1 allows for an elevated role of factors. Indeed, Caselli shows that if the ES can be as low as 0.5 the *entire* variation in GDP per worker can be accounted for by factors. This is in itself surprising: when Harrod neutral technological change is assumed an ES *above* 1 raises the impact from factors. We clarify the reason for the apparent "reversal" of the impact from ES on the accounting results, and proceed to revisit our two tests in light of this possible "world view".

If indeed technology is simultaneously Harrod and Solow neutral, the simple tests mentioned above fail to convey accurate information about the size of the ES. However, assuming the tests fail to identify the size of the ES, one can demonstrate that the information they do convey is nevertheless sufficient to provide *a lower bound* on the ES.[3] To be specific, we find that if technology enters the production function simultaneously in a Harrod and Solow neutral fashion the lower bound for the ES is 0.77. As shown below, if the ES is bounded from below by 0.77, the results from performing development accounting with the simpler CD specification will not yield misleading results, *even if* the more appropriate assumption were a CES specification with Harrod and Solow neutral technology.

Taken together therefore these results provide a strong case that, for the purpose of development accounting, using an aggregate CD production function is a reasonable shortcut to using a more general CES production function. With a more general CES function the bias of technological change becomes a meaningful concept. Moreover, the nature of the bias (Harrod, Solow etc.) inevitably impinges upon what is a reasonable assumption for the ES, given observations on capital-output ratios, labor shares etc. The tests and calibrations of ES we perform tell us, however, that no matter what the true bias of technological change is, the relevant ES will always be of a size such that the CD approximation is reasonably accurate in the context of development accounting. Thus, even though we do not know a priori whether technological change actually manifests itself as Harrod neutral, Solow neutral or both, nonetheless the conclusion emerges: it is OK to assume that the world is Cobb–Douglas when accounting for productivity.

The paper proceeds as follows. The next section lays out the consequences, for the result stemming from development accounting, of employing a general CES function where technological change is Harrod neutral. Section 3 presents evidence on the empirical relationship between factor shares and capital-output ratios, and Section 4 shows our calibration of the elasticity of substitution. Section 5 discusses the implications, for accounting and our tests, of simultaneously allowing for Harrod and Solow neutral technological change, and provides a range of ES consistent with the new technology assumption and data on relative shares and capital-output ratios. Section 6 discusses our results, and relates them to previous findings. Finally Section 7 concludes.

2. Preliminaries: development accounting with a CES Production function

Consider the following specification for the aggregate production function

$$Y = \begin{cases} \left[\alpha K^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(AhL)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} & \text{if } \epsilon \neq 1 \\ K^{\alpha}(AhL)^{1-\alpha} & \text{if } \epsilon = 1, \end{cases}$$

[1] It is worth remarking that the issue of the size of ES is important in other contexts than development accounting. In general the ES matters for whether endogenous growth is feasible or not (e.g., Pitchford, 1960), whether multiplicity of steady state equilibria can emerge or not (e.g., Galor, 1996) and whether standard endogenous growth models feature scale effects or not (Dalgaard and Jensen, 2007).

[2] Again, under the standard assumptions of development accounting we would have to reject that the production function is *exactly* CD, as it would imply zero variation in relative factor-shares, which is counterfactural.

[3] As demonstrated by Diamond et al. (1978), if the direction of the bias in technological change is unknown the other technological parameters (e.g., the ES) cannot be identified in general. The theorem does not rule out, however, that a lower bound on the ES can be established.