Historia Mathematica **27** (2000), 171–201
doi:10.1006/hmat.1999.2274, available online at http://www.idealibrary.com on IDEAL®

Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World

Alpay Özdural

Faculty of Architecture, Eastern Mediterranean University, Gazi Magusa P.O. Box 95, Mersin 10, North Cyprus, Turkey

E-mail: alpay@sinan.emu.edu.tr

Two mathematical sources, *On the Geometric Constructions Necessary for the Artisan*, by Abu'l-Wafā' (ca. 940–998), and the anonymous work, *On Interlocks of Similar or Corresponding Figures* (ca. 1300), provide us with insight into the collaboration between mathematicians and artisans in the Islamic world. In this paper I present a series of quotations from these two sources, which show that mathematicians taught geometry to artisans by means of cut-and-paste methods and of geometrical figures that had the potential of being used for ornamental purposes. © 2000 Academic Press

Matematikle ilgili iki kaynak bize İslam dünyasında matematikçiler ile sanatkarlar arasındaki işbirliği konusunda aydınlatıcı bilgiler sunuyor. Bu kaynaklardan biri Abu'l-Vefa (ca. 940–998) tarafından yazılan "Sanatkarın ihtiyaç duyduğu geometrik çizimler," diğeri anonim bir yazarın kaleme aldığı "İçiçe geçen benzer veya karşılıklı şekiller" (ca. 1300). Bu iki kaynaktan derlediğim bir dizi alıntıya yer verdiğim bu makalede görüyoruz ki matematikçiler sanatkarlara kes-ve-yapıştır yöntemiyle geometri öğretirken, aynı zamanda önerdikleri geometrik şekillerin bezeme sanatlarında kullanılabilir olmasına özen gösteriyorlardı. © 2000 Academic Press

MSC subject classifications: 01A30, 01A20.
Key Words: Abu'l-Wafā'; Islamic geometry; Islamic art; mosaics; Abū Bakr al-Khalīl.

INTRODUCTION

Intriguing patterns ornamenting architectural monuments and other objects of art bear witness to the predominance of geometry in Islamic art. Traditionally, the artisans who produced them were believed to be experts in geometry. Recent studies, however, have shown that mathematicians who taught practical geometry to artisans played a decisive role in the creation of those patterns and perhaps in designing the buildings themselves. Medieval Islamic *mathematical* literature is thus our main source for the relation between mathematics and the architectural arts in medieval Islam [20–23; 27; 37–41].

Abu'l-Wafā' al-Būzjānī (940–ca. 998)[1] tells us in his book, *On the Geometric Constructions Necessary for the Artisan* (Kitāb fīmā yaḥtāju ilayhi al-ṣānic min al-aʿmāl al-handasiya; hereinafter *Geometric Constructions*), that he attended meetings between geometers and artisans in Baghdād [4, 53].[2] Such meetings were a widespread phenomenon in the Islamic world. In 11th- century Isfahan, for example, ʿUmar Khayyām solved a right triangle with

[1] On Abu'l-Wafā', see [46, 321–325].

[2] For my study, I have used the manuscript Istanbul, Ayasofya 2753 (a facsimile of this manuscript is in [42]). It was copied in the first half of the 15th century for Ulugh Beg's library in Samarkand and was brought to Istanbul by ʿAlī Kuşçu in 1471, when he settled and started teaching in the Ayasofya Madrasa there. This manuscript is

0315-0860/00 $35.00

the aid of a cubic equation in an untitled treatise. He explained that the treatise was prompted by a question at a meeting, which seems to have been attended by artisans and geometers [12; 31; 34; 43]. In the 15th century, Ghiyāth al-Dīn Jamshīd al-Kāshī solved a problem about a triangular leveling instrument at the construction site of the astronomical observatory in Samarkand during a meeting of artisans, mathematicians, and other dignitaries [29, 101]. We learn from Ca'fer Efendi that meetings between architects and geometers were held frequently in Istanbul in the late 16th and early 17th centuries [16, 28]. Ca'fer Efendi reportedly compiled a treatise from his notes taken at such meetings over a period of 20 years [16, 22–23], although no copies of this treatise have yet been found.

Here, I discuss an anonymous Persian work on ornamental geometry, which may be similar to the lost work by Ca'fer Efendi, and which seems to have been compiled from notes taken by a scribe at a series of meetings between geometers and artisans.[3] The title of the work appears as a vertical marginal note: *Interlocks of Similar or Corresponding Figures* (Fī tadākhul al-ashkāl al-mutashābiha aw al-mutawāfiqa; hereinafter *Interlocks of Figures*) [11]. This work can be dated to around 1300 because the mathematician, Abū Bakr al-Khalīl al-Tājir al-Raṣadī (ca. 1300),[4] is cited twice as one of the participants of the discussions, and the text probably came from Tabrīz.[5]

The mathematicians seemed pleased to be introduced to a field full of rewarding concrete applications. For instance, 'Umar Khayyām says, "If it were not for the highness of this meeting, may its highness last forever, and for the appropriateness of the proposer of the question, may God bless him, I would have been far away from this field" [12, 336; 43, 90]. It is thus conceivable that some of the aesthetic, spatial, and structural innovations of Islamic architecture are due to mathematicians.[6]

The meetings between mathematicians and artisans were normally not documented, but fortunately there are exceptions. The account by Abu'l-Wafā' in *Geometric Constructions* and the descriptions in *Interlocks of Figures* provide insights into how mathematicians and artisans collaborated. At some sessions, mathematicians gave instructions on certain principles and practices of geometry. At others, they worked on geometric constructions of two- or three-dimensional ornamental patterns or gave advice on the application of geometry to architectural construction. Below, I argue that the mathematicians used cut-and-paste methods as a didactical tool in teaching geometry to artisans. By presenting literal

dedicated to the Buyid ruler Al-Manṣūr Bahā' al-Dawla, who is referred to by the title Shāhinshāh. This title could have been used only for the rulers of Persia, and Al-Manṣūr was the ruler of Persia from A.D. 998 to 1013. Therefore, the text was probably compiled by a student of Abu'l-Wafā' after the death of his master in 998. This argument can be supported by the fact that passages such as "Abu'l-Wafā' (or the professor or the sage) said" appear in the text. For other manuscript copies, commentaries, and modern translations of Abu'l-Wafā's text, see [1–3; 5–8; 14–15; 32–33; 42; 47–48].

[3] In [40], I have argued that this treatise was a compilation of notes taken over a period of time.

[4] According to the information kindly provided by Mohammad Bagheri and Mehran Akhbarifar (both of Tehran), Abū Bakr al-Khalīl is cited as the father of the copyist of a manuscript in Mashhad. Since the manuscript was copied in 1327–1328, Abū Bakr al-Khalīl must have been active around 1300. The only other available information about this mathematician is a remark found at the end of an anonymous undated Persian treatise (in the same Paris manuscript as *Interlocks of Figures*), praising him as "the pride of geometers" [10, 118v].

[5] The provenance of *Interlocks of Figures* and the milieu in which it was written will be discussed below, and in my English translation with commentary, to appear in due course. For modern studies on this work, see [13, 315–340; 15, 73–95; 17–19; 35, 129–181; 40].

[6] In [41], I have argued that 'Umar Khayyām was the designer of the North Dome Chamber of the Friday Mosque of Isfahan, which is celebrated for the maturity and elegance of its proportions.