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An efficient genetic algorithm for determining the optimal price discrimination

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Abstract

This paper proposes a genetic algorithm (GA) to solve a price discrimination model developed to determine the product's selling price in two markets. The model considers the production as a function of price in one market and as a function of price and marketing expenditure in other market. The cost of production is also assumed to be a function of production in both markets. The objective is to maximize the benefit under restricted prices and marketing expenditure in two markets. The considered model is a posynomial geometric programming (GP) problem with continues non-linear/non-convex objective function and linear boundary constraints. The proposed model cannot be solved optimally by a common approach such as geometric programming in a reasonable amount of time. The results show that the proposed GA can become rapidly convergent to near global optimum. Some of typical numerical examples are solved by GA and results are reported. Also, we implement a sensitivity analysis on some of model parameters in order to have a better understanding of the performance and behavior of the proposed GA.

Keywords: Geometric programming; Genetic algorithm; Optimization; Production and operation management; Economics

1. Introduction

One of the most important issues on having a fair price discrimination strategy is on choosing a right model. Many traditional discrimination models assume production as a function of price. The production function often is in a form of linear or quadratic. On the other hand, economists normally study the cost of production as a function of production with similar linear or quadratic pattern. The production and cost function considered, in this paper, have an exponential form of price and production, respectively. This type of modeling has been widely used in the literature [1–5]. They consider demand as a function of price and marketing expenditure and assume that when demand increases production will be less costly. Sadjadi et al. [5] studies the effects of integrated production and marketing decisions in a profit maximizing firm. Their model can determine price, marketing expenditure, demand or production volume; and lot size for a single product with stable demand when economies of scale are given. Their model is a signomial geometric programming

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(GP) problem with the degree of difficulty 1. Therefore, they use GP to locate the optimal solution of the proposed model. Lee [2] considers the same demand function for determining order quantity and selling price. In their implementation, they use a previous [3] model formulation, with an adaptation of GP, to determine the global solution of the model. The primary objective is to determine price and marketing strategies in two markets. We assume there is no competition in the first market. Therefore there is no need for any advertisement on selling goods. In other word, pricing strategy is the only way to promote market. On the other hand, we plan to penetrate into a highly competitive market in the second market not only by considering pricing strategy but by having a good marketing strategy. The objective function of the proposed model is to maximize the total profit in two markets which includes marketing expenditure in the second market. We consider three boundary constraints for decision variables of the proposed model to limit the solution space. These boundaries can be experimentally determined by an expert's knowledge. The resulted problem is in posynomial GP problem [6]. We use genetic algorithm (GA) method to find a neighbor of global minimum of the resulting model. Sensitivity analysis by GA is presented in order to analyze the behavior of the proposed method under different conditions and also verify the performance of the proposed GA.

Genetic algorithms (GAs) [7] belong to meta-heuristic approaches category that attempt to mimic the biological evolution process for discovering good solutions. They are based on a direct analogy to the Darwinian natural selection and mutations in the biological reproduction. They belong to a category of heuristics known as stochastic methods employing randomized choice operators in their search strategy and do not depend on priori knowledge of the features of the domain completely. These operators have been conceived through abstractions of natural genetic mechanisms such as crossover and mutation and they have been cast into algorithmic forms. Repetitive executions of these heuristics need not yield the same solution. A genetic algorithm maintains a collection or population of solutions throughout the search. It initializes the population with a pool of potential solutions to the problem and seeks to produce better solutions (individuals) by combining the better of the existing ones through the use of one or more genetic operators. Individuals are chosen at each iteration with a bias towards those with the best objective or fitness values. With various mapping techniques and an appropriate measure of fitness of individuals (i.e., objective function value), a genetic algorithm can be tailored to evolve a solution for many types of discrete/continues models. Although meta-heuristics have been widely used for many mathematical models, to the best authors' knowledge, no one has ever used GA to solve GP models. Only Wang et al. [8] uses a hybrid of GA and simulated annealing (GSA) to select the optimal machining parameters for multi-pass milling. They compare the performance of proposed GSA with a GP and dynamic programming (GP + DP) based method and show that the GSA is more effective for optimizing the cutting parameters for milling operation than conventional GP + DP [8].

This paper is organized as follows. We first present the problem description. Next, GA method is used to find the optimal solution of the problem formulation. Computational results and sensitivity analysis is then presented in the following section to derive some managerial implications. 'Throughout the paper, we use some numerical examples in order to show the implementation of the algorithm and analyze the behavior of the parameters of our model. Finally, conclusion remarks are presented at the end to summarize the contribution of the work.

2. Problem description

Consider a single product where demand is affected by selling price. Let P, α, M and γ be the selling price per unit, price elasticity to demand, marketing expenditure per unit and marketing expenditure elasticity to demand, respectively. We assume that the production volume must be determined in two markets. In the first one, we are the only provider of the product and in the second market place we have to compete with the others through price and marketing planning. For the first market we assume,

$$Q_1 = k_1 P_1^{-\alpha_1} \tag{1}$$

and

$$Q_2 = k_2 P_2^{-\alpha_1} M^{\gamma}, \tag{2}$$

where production lot size in market 1, Q_1, is defined as a function of price per unit in market 1, P_1, and production lot size in market 2, Q_2, is defined as a function of price per unit in market 2, P_2 and marketing cost in