ELSEVIER

Int. J. Production Economics 65 (2000) 201–216

international journal of production economics

www.elsevier.com/locate/dsw

Optimal ordering, discounting, and pricing in the single-period problem

Moutaz J. Khouja*

Information & Operations Management Department, The Belk College of Business Administration, The University of North Carolina at Charlotte, Charlotte, NC 28223, USA

Received 22 October 1998; accepted 23 March 1999

Abstract

The single-period problem (SPP), also known as the newsboy or newsvendor problem, is to find the order quantity which maximizes the expected profit in a single-period probabilistic demand framework. Previous extensions to the SPP include, in separate models, the simultaneous determination of the optimal price and quantity when demand is price-dependent, and the determination of the optimal order quantity when progressive discounts with preset prices are used to sell excess inventory. In this paper, we extend the SPP to the case in which demand is price-dependent and multiple discounts with prices under the control of the newsvendor are used to sell excess inventory. First, we develop two algorithms for determining the optimal number of discounts under fixed discounting cost for a given order quantity and realization of demand. Then, we identify the optimal order quantity before any demand is realized. We also analyze the joint determination of the order quantity and initial price. We illustrate the models and provide some insights using numerical examples. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Inventory management; Production management; Operations management

1. Introduction

The classical single-period problem (SPP) is to find a product's order quantity which maximizes the expected profit in a probabilistic demand framework. The SPP model assumes that if any inventory remains at the end of the period, one discount is used to sell it or it is disposed of [1]. If the order quantity is smaller than the realized demand, the newsvendor, hereafter NV, forgoes some profit. If the order quantity is larger than the realized demand, the NV losses some money because he/she has to discount the remaining inventory to a price below cost. The SPP is reflective of many real-life situations and is often used to aid decision making in the fashion and sporting industries, both at the manufacturing and retail levels [2]. The SPP can also be used in managing capacity and evaluating advanced booking of orders in service industries such as airlines, hotels, etc. [3].

Several researchers have suggested SPP extensions in which demand is price dependent [4–10]. Whitin [4] assumed that the expected demand is a function of price and using incremental analysis, derived the necessary optimality condition. He then

* Tel.: 001-704-547-3242; fax: 001-704-547-3123.
E-mail address: mjkhouja@email.uncc.edu (M.J. Khouja)

0925-5273/00/$ - see front matter
PII: S0925-5273(99)00027-4

provided closed-form expressions for the optimal price, which is used to find the optimal order quantity for a demand with a rectangular distribution. Mills [5] also assumed demand to be a random variable with an expected value that is decreasing in price and with constant variance. Mills derived the necessary optimality conditions and provided further analysis for the case of demand with rectangular distribution.

Lau and Lau (LL) [6] introduced a model in which the NV has the option of decreasing price in order to increase demand. LL analyzed two cases for demand:

(a) *Case A*: Demand is given by a simple homoscedastic regression model $x = a - bP + \varepsilon$, where a and b are constants, x is the quantity demanded, P is unit price, and ε is normally distributed. The above equation implies a normally distributed demand with an expected value which decreases linearly with unit price.

(b) *Case B*: Demand distribution is constructed using a combination of statistical data analysis and experts' subjective estimates. The 'method of moments' was used to fit the four-parameter beta distribution to estimate demand.

For case A, LL showed that the expected profit is unimodal and thus the golden section method can be used for maximization. For case B, there is no guarantee the expected profit is unimodal. Thus, LL developed a search procedure for identifying local maximums. LL also solved the problem under the objective of maximizing the probability of achieving a target profit and considered both zero and positive shortage cost cases. For zero shortage cost and demand given by case A, LL derived closed-form solutions for the optimal order quantity and optimal price. For zero shortage cost and demand given by case B, LL developed a procedure for computing the probability of achieving a target profit and used a search procedure for finding a good solution. For positive shortage cost and demand given by cases A or B, the probability of achieving a target profit may not be unimodal. LL developed procedures for computing the probability of achieving a target profit and identifying a good solution.

Polatoglu [7] also considered the simultaneous pricing and procurement decisions. Polatoglu identified few special cases of the demand process addressed in the literature: (i) an additive model in which the demand at price P is $x(P) = \mu(P) + \varepsilon$, where $\mu(P)$ is the mean demand as a function of price, and ε is a random variable with a known distribution and $E[\varepsilon] = 0$, (ii) a multiplicative model in which $x(P) = \mu(P)\varepsilon$ where $E[\varepsilon] = 1$, (iii) a riskless model in which $X(P) = \mu(P)$. Polatoglu analyzed the SPP under general demand uncertainty to reveal the fundamental properties of the model independent of the demand pattern. Polatoglu assumed an initial inventory of I, $\mu(P)$ is a monotone decreasing function of P on $(0, \infty)$, and a fixed ordering cost of k. For linear expected demand, ($\mu(P) = a - bP$, where $a, b > 0$) Polatoglu proved the unimodality of the expected profit for uniformly distributed additive demand and exponentially distributed multiplicative demand.

Khouja [8] solved an SPP in which multiple discounts are used to sell excess inventory. In this model, retailers progressively increase the discount until all excess inventory is sold. The product is initially offered at the regular price P_0. After some time, if any inventory remains the price is reduced to $P_1, P_0 > P_1$. In general, the prices are P_i, $i = 0, 1, \dots, n$, where $P_i > P_{i+1}$. The amount demanded at each P_i is assumed to be a multiple t_i, $i = 1, \dots, n$ of the demand at the regular price P_0. Khouja solved the problem under two objectives: (a) maximizing the expected profit, and (b) maximizing the probability of achieving a target profit. Khouja showed that the expected profit is concave and derived the sufficient optimality condition for the order quantity. For maximizing the probability of achieving a target profit, Khouja provided closed-form expression for the optimal order quantity. Khouja [10] developed an algorithm for identifying the optimal order quantity for the multi-discount SPP when the supplier offers the NV an all-units quantity discount. Khouja and Mehrez [9] provided a solution algorithm to the multi-product multi-discount constrained SPP.

The above models may not capture some actual problems facing many NVs. While NV may consider the demand–price relationship is determining the order quantity, he/she still faces the problem of what to do with excess inventory when the order