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Original Articles

Direct and indirect influences of executive functions on mathematics achievement

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ABSTRACT

Achievement in mathematics is predicted by an individual's domain-specific factual knowledge, procedural skill and conceptual understanding as well as domain-general executive function skills. In this study we investigated the extent to which executive function skills contribute to these three components of mathematical knowledge, whether this mediates the relationship between executive functions and overall mathematics achievement, and if these relationships change with age. Two hundred and ninety-three participants aged between 8 and 25 years completed a large battery of mathematics and executive function tests. Domain-specific skills partially mediated the relationship between executive functions and mathematics achievement: Inhibitory control within the numerical domain was associated with factual knowledge and procedural skill, which in turn was associated with mathematical achievement. Working memory contributed to mathematics achievement indirectly through factual knowledge, procedural skill and, to a lesser extent, conceptual understanding. There remained a substantial direct pathway between working memory and mathematics achievement however, which may reflect the role of working memory in identifying and constructing problem representations. These relationships were remarkably stable from 8 years through to young adulthood. Our findings help to refine existing multi-component frameworks of mathematics and understand the mechanisms by which executive functions support mathematics achievement.

1. Introduction

A good understanding of mathematics is essential for success in modern society, leading not only to good job prospects but also a better quality of life (Gross, Hudson, & Price, 2009; OECD, 2013; Parsons & Bynner, 2005). Children develop an understanding of mathematics throughout their primary and secondary education. In order to ensure effective pedagogy that supports the needs of all learners it is critical to recognise the range of factors that contribute to mathematical achievement so that teaching practices can be targeted appropriately. One set of factors that play an important role in mathematics achievement are the cognitive resources that an individual can draw on. Here we evaluate the direct contribution of domain-general skills, in particular executive functions, the set of processes that control and guide our information processing, to mathematics achievement. In addition we explore to what extent the contribution of executive functions to mathematics achievement is mediated by domain-specific mathematical abilities, and whether this changes with age. Addressing these questions will refine our understanding of the ways in which executive functions support mathematics achievement, which can then inform intervention approaches that aim to capitalise on this relationship.

Attainment in mathematics rests on success in a number of underlying cognitive skills. Several researchers have proposed a multi-component model in which mathematics is underpinned by both domain-specific mathematical knowledge in addition to more general cognitive processes (Fuchs et al., 2010; Geary, 2011; LeFevre et al., 2010). For example, Le Fevre's Pathways Model of early mathematical outcomes includes linguistic and spatial attention pathways in addition to a quantitative pathway. Geary (2004; Geary & Hoard, 2005) outlined a hierarchical framework (see Fig. 1) in which achievement in any area of mathematics is underpinned by skill in applying the appropriate procedures, and an understanding of the underlying concepts. In turn, these domain-specific processes draw upon a range of domain-general skills, including language and visuospatial skills and in particular executive functions. This model therefore suggests that the

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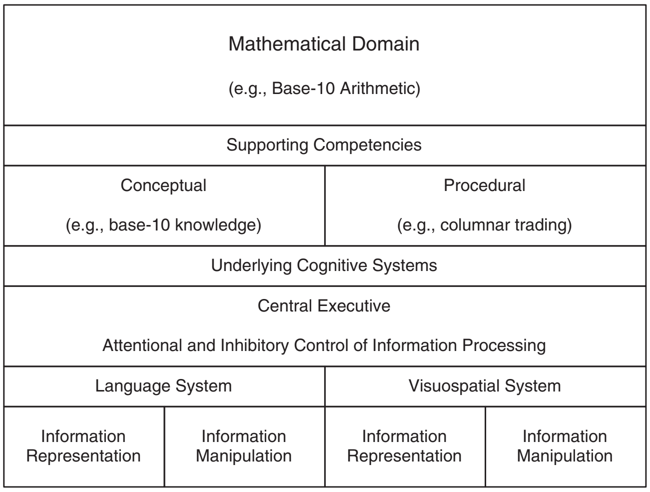

Fig. 1. Hierarchical framework of the skills underpinning mathematics. Taken from Geary (2004).

influence of executive function skills on mathematics achievement is mediated through its role in domain-specific mathematical competencies.

It is well established that an individuals' procedural skill and conceptual understanding contribute to their mathematical achievement, in addition to their factual knowledge: the ability to recall stored number facts from long-term memory (Baroody, 2003; Cowan et al., 2011; Dowker, 2005; Hiebert, 1986; LeFevre et al., 2006). More recently, a growing body of evidence has demonstrated a link between domain-general executive functions and mathematics achievement (see Bull & Lee, 2014; Cragg & Gilmore, 2014 for reviews). Executive functions, the skills used to guide and control thought and action, are typically divided into three main components following Miyake et al. (2000). These are (i) *updating* or *working memory*, the ability to monitor and manipulate information held in mind, (ii) *inhibition*, the suppression of irrelevant information and inappropriate responses, and (iii) *shifting*, the capacity for flexible thinking and switching attention between different tasks. Below we review the literature exploring the links between each of these components of executive functions and overall mathematics achievement before going on to consider its contribution to the underpinning skills of factual knowledge, procedural skill and conceptual understanding.

1.1. Executive functions and mathematics achievement

Across many studies working memory has been found to be a strong predictor of mathematics outcomes, both cross-sectionally (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013) and longitudinally (Fuchs et al., 2010; Hecht, Torgesen, Wagner, & Rashotte, 2001). According to the influential Baddeley and Hitch (1974) model of working memory, adopted by the majority of researchers in this field, working memory is made up of short-term stores for verbal and visuospatial information in addition to a central executive component that coordinates these storage systems and allows the manipulation and storage of information at the same time. Accordingly, tasks that simply require information to be stored for a short amount of time are used as an index of the capacity of the verbal and visuospatial stores, while tasks that require the simultaneous storage and manipulation of information are used to also tap into the central executive component of working memory. In general, tasks that tap into this executive working memory system show stronger relationships with mathematics achievement than those which simply measure the short-term storage of information. The results from a recent meta-analysis of 111 studies found that verbal executive working memory showed the strongest relationship with mathematics, followed by visuospatial executive working memory and short-term storage, which did not differ, and finally the short-term storage of verbal information (Friso-van den Bos et al., 2013). This suggests that it is the central executive component of working memory that is most important for mathematics.

The tasks that are typically used to tap into the central executive are not a pure measure of this process however, as the short-term storage and processing of information is also required. To try and isolate the exact components of working memory that contribute to mathematics achievement Bayliss and colleagues adopted a variance partitioning approach whereby they used a complex span combining the storage and processing of information, as typically used to index executive working memory, but also measured storage and processing independently. Using a series of regression models they were able to isolate the unique variance associated purely with the central executive, storage capacity and processing speed, as well as the shared variance between these processes. In one study with 7–9-year-olds, Bayliss, Jarrold, Gunn, and Baddeley (2003) found that the executive demands of combining verbal storage and processing explained significant variance in mathematics achievement, but that combining visuospatial storage and processing did not. Moreover, the executive working memory tasks involving verbal storage explained more variance in mathematics achievement than a short-term verbal storage task alone.

A follow-up study investigating developmental changes in working memory and cognitive abilities (Bayliss, Jarrold, Baddeley, Gunn, & Leigh, 2005) demonstrated that shared variance between age, working memory, storage and processing speed across both verbal and visuospatial domains contributed most to mathematics achievement across ages, explaining 38% of the variance. The central executive accounted for around 5% of unique variance, as did shared variance between age, working memory and storage. Storage alone accounted for 2.5% of the variance, which was attributed to variation in the ability to reactivate items in memory. Processing speed accounted for a small amount of variance both uniquely (1.3%) and shared with working memory and age (25%). Taken together, these findings suggest that all components of working memory play some role in successful mathematics achievement but that the demands of combining the storage of verbal information with additional information processing do seem to be particularly important for mathematics achievement in childhood.

The findings of Friso-van den Bos et al. and Bayliss et al. suggest that there may be some domain-specificity in the relationship between working memory and mathematics achievement, with verbal working memory playing a larger role than visuospatial working memory. Other researchers have argued for the opposite pattern however, with a stronger relationship between mathematics and visuospatial working memory than verbal working memory, particularly in children with mathematics difficulties but with typical reading and/or verbal performance (Andersson & Östergren, 2012; McLean & Hitch, 1999; Schuchardt, Maehler, & Hasselhorn, 2008; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2013). In a comprehensive study which tested a large sample of typically developing 9-year-olds on an extensive battery of measures, Szűcs, Devine, Soltesz, Nobes, and Gabriel (2014) found that visuospatial short-term and working memory were significant predictors of mathematical achievement, while verbal short-term and working memory were not. Phonological decoding and verbal knowledge were found to be significant predictors however, which may have accounted for some of the variance associated with verbal short-