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Are European equity markets efficient? New evidence from fractal analysis

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ABSTRACT

We report an empirical analysis of long-range dependence in the returns of eight stock market indices, using the Rescaled Range Analysis (RRA) to estimate the Hurst exponent. Monte Carlo and bootstrap simulations are used to construct critical values for the null hypothesis of no long-range dependence. The issue of disentangling short-range and long-range dependence is examined. Pre-filtering by fitting a (short-range) autoregressive model eliminates part of the long-range dependence when the latter is present, while failure to pre-filter leaves open the possibility of conflating short-range and long-range dependence. There is a strong evidence of long-range dependence for the small central European Czech stock market index PX-glob, and a weaker evidence for two smaller western European stock market indices, MSE (Spain) and SWX (Switzerland). There is little or no evidence of long-range dependence for the other five indices, including those with the largest capitalizations among those considered, DJIA (US) and FTSE350 (UK). These results are generally consistent with prior expectations concerning the relative efficiency of the stock markets examined.

1. Introduction

Traditional capital markets theory relies on the assumption that logarithmic prices are martingales. Accordingly, logarithmic returns are temporally independent. Temporal dependence in returns is inconsistent with the Efficient Market Hypothesis (EMH) (Fama, 1970), mean-variance portfolio theory (Markowitz, 1952, 1959), and the capital asset pricing model (CAPM) (Sharpe, 1964; Lintner, 1965). While short-range dependence is unlikely to provide a basis for the development of trading strategies that deliver positive abnormal returns consistently, long-range dependence, under certain conditions, implies that trading strategies based on historical prices may be systematically profitable (Mandelbrot, 1971; Rogers, 1997).

In order to allow for temporal dependence in returns, and other stylized facts concerning stock market behavior, Peters (1994) introduces the Fractal Markets Hypothesis (FMH). The FMH does not reject *a priori* the assumption that returns are independent, but it does allow for a broader range of behavior. Accordingly, the FMH does not necessarily constitute an alternative to the EMH, but rather a generalization. The FMH derives from the theory of fractals (Mandelbrot, 1982). A fractal is an object whose parts resemble the whole. Peters (1994) suggests that financial markets have a fractal structure: when markets are stable, returns calculated over different time scales (daily, weekly, monthly, and so on) exhibit the same autocovariance structure. For instance, if daily returns exhibit positive temporal dependence, so do weekly and monthly returns. This feature is called self-affinity.

In the case where the variance and higher-order moments of the returns series are finite, the autocovariance structure of a self-affine time series is represented by the Hurst exponent. A Hurst exponent of 0.5 indicates that returns measured over any time scale are random. A Hurst exponent larger (smaller) than 0.5 indicates positive (negative) long-range dependence in returns measured over any time scale. In the case of positive long-range dependence, the autocorrelation function decays slowly. The Rescaled Range Analysis (RRA) has been employed widely to calculate the Hurst exponent, interpreted as an indicator of long-range dependence.

Empirical studies of long-range dependence in financial returns series precede the FMH (Peters, 1994). Greene and Fielitz (1977) and Peters (1991) report evidence of long-range dependence in US stock market returns. Subsequent refinements of the methodology used to measure long-range dependence produce results consistent with the EMH (Lo, 1991). Recently, Serletis and Rosenberg (2009) fail to find evidence of long-range dependence for four US stock market indices. A number of studies have examined international stock markets (Cheung & Lai, 1995; Jacobsen, 1996; Opong, Mulholland, Fox, & Farahmand, 1999; Howe, Martin, & Wood, 1999; McKenzie, 2001; Costa & Vasconcelos, 2003; Kim & Yoon, 2004; Zhuang, Huang, & Sha, 2004; Norouzzadeh & Jafari, 2005; Onali & Goddard, 2009); commodities markets (Cheung & Lai, 1993; Alvarez-Ramirez, Cisneros, Ibarra-Valdez, & Soriano, 2002; Serletis & Rosenberg, 2007); and exchange rates (Mulligan, 2000; Kim & Yoon, 2004; Da Silva, Matsushita, Gleria, & Figueiredo, 2007). Grech and Mazur (2004) and Grech and Pamula (2008) examine the connection between the

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Hurst exponent and stock market crashes. In a cross-country analysis, Cajueiro and Tabak (2004, 2005) interpret estimated Hurst exponents for either stock returns or volatility as indicators of stock market efficiency. There is a lack of consensus in this literature as to the most appropriate method for measuring long-range dependence. Moreover, in the absence of a sound methodology for statistical inference, many studies base their conclusions solely on point estimates of the Hurst exponent.

This paper reports tests for the validity of the EMH for eight stock markets that are believed to be at different stages of development. We examine stock market indices comprising of large numbers of stocks. Predictability of returns owing to thin trading is unlikely. Rejection of the null hypothesis of no long-range independence would suggest the possible existence of arbitrage opportunities.

We contribute to the extant literature in several ways. First, we provide strong evidence of departure from random walk behavior in logarithmic prices, in the form of long-range dependence in logarithmic returns, for the Czech stock market index PX-Glob. Weaker evidence of long-range dependence is found for the Spanish and Swiss market indices, MSE and SWX. For the other five market indices examined, there is little or no evidence of long-range dependence in returns.

Second, unlike most of the extant literature, we employ Monte Carlo simulations to construct critical values for the null hypothesis that returns are normal and independent and identically distributed (NIID), and bootstrap simulations to construct critical values for the null hypothesis that returns are independent and identically distributed (IID). The Monte Carlo simulations require a normality assumption, while the bootstrap simulations assume that the observed values of each returns series are representative of its underlying distributional properties.

Third, it is widely recognized that the identification of long-range dependence in the presence of short-range dependence is challenging, owing to difficulties in disentangling the short and long memory components (Smith, Taylor, & Yadav, 1997). In some previous studies, the RRA is applied to the residuals of a fitted autoregressive model for the returns series, to eliminate any short-range dependence that may be present by pre-filtering before testing for long-range dependence (Peters, 1994; Jacobsen, 1996; Opong et al., 1999). Alternative methods to control for short-range dependence are suggested by Lo (1991) and Fillol and Tripier (2004).

We run the RRA on the original series both with and without pre-filtering. We compare the estimated Hurst exponent for pre-filtered returns with critical values constructed using NIID Monte Carlo and IID bootstrap simulations; and we compare the estimated Hurst exponent for unfiltered returns with critical values constructed using recursive Monte Carlo and recursive bootstrap simulations, in which the simulated series have a short-range dependence structure that corresponds to a fitted autoregressive model for the actual returns series for each index.

The rest of the paper is structured as follows. Section 2 reviews the properties of self-affinity, long-range dependence, and the generalized Central Limit Theorem. Section 3 describes the methodology and data. Section 4 reports the empirical results. Section 5 concludes.

2. Self-affinity and long-range dependence

Self-similarity is the distinguishing feature of fractals: each part comprising a fractal resembles the whole. The weaker concept of self-affinity is usually employed in the case of a financial asset (logarithmic) returns series. After rescaling by applying a factor that depends only on the time scale (or frequency) over which the returns are measured (for example, daily, weekly, monthly or quarterly), a self-affine returns series has the same distributional properties for every time scale. Self-affinity implies that '[…] the distribution of returns over different sampling intervals is identical except for a single, non-random contraction' (Mandelbrot, Fisher, & Calvet, 1997, p. 8).

Self-affinity is described as follows (Calvet & Fisher, 2002, p. 383):

$$\left\{X(nt_1),\dots,X(nt_k)\right\} \overset{d}{=} \left\{n^H X(t_1),\dots,n^H X(t_k)\right\} \quad (1)$$

where X(t) denotes the (logarithmic) return measured over the period t, $H>0$, $n,k,t_1,\dots t_k \geq 0$, and $\overset{d}{=}$ denotes equality in distribution.

In the case of a normally distributed returns series, the property of self-affinity implies that the variance of the returns, denoted by $\gamma_0^{(n)}$, varies in proportion to the scale over which the returns are measured, n, according to a factor of proportionality governed by the Hurst exponent, H:

$$\gamma_0^{(n)} = n^{2H}\gamma_0^{(1)}. \quad (2)$$

The property of self-affinity implies that the autocorrelation function for lag k, $\rho_k^{(n)} = \gamma_k^{(n)} / \gamma_0^{(n)}$, is identical for all n, or $\rho_k^{(n)} = \rho_k^{(1)}$ for $n\geq 1$ and $k\geq 1$:

$$\gamma_k^{(n)} = (1/2)\left[(k+1)^{2H} - 2k^{2H} + |k-1|^{2H}\right]. \quad (3)$$

A fractionally-integrated returns series, which gives rise to a price series characterized as Fractional Brownian Motion (FBM), satisfies the property of self-affinity. FBM is a generalization of Brownian Motion, the continuous-time analog of the random walk. FBM has similar features to Brownian Motion, but with increments that are long-range dependent and therefore non-random (Mandelbrot & van Ness, 1968). Long-range dependence (at all time scales) in a self-affine returns series is represented by the parameter $0<H<1$. For $0<H<0.5$, the values of the series k periods apart are negatively correlated for all k. For $0.5<H<1$, values of the series k periods apart are positively correlated, or long-range dependent. The Efficient Market Hypothesis (EMH) is violated for any $H\neq 0.5$. An extensive literature examines whether the EMH correctly represents the behavior of stock returns, using fractal analysis. If the EMH is rejected, the FMH provides an alternative account of the behavior of returns.

The best-known discrete-time representation of long-range dependence is provided by the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model (Granger & Joyeux, 1980; Hosking, 1981). An ARFIMA(0,d,0) process, where $d = H - 0.5$, is asymptotically self-affine (Fisher, Calvet, & Mandelbrot, 1997). The ARFIMA(p,d,q) model accommodates both short-range dependence and long-range dependence. Owing to the presence of short-range dependence, however, estimation of the Hurst exponent by applying the RRA to an ARFIMA(p,d,q) series is unreliable. Jacobsen (1996) and Fillol and Tripier (2004) report that pre-filtering to eliminate short-range dependence reduces the power of the test for long-range dependence.

Lo (1991) adjusts the original RRA to allow for short-range dependence, by including weighted autocovariances for a selected number of lagged returns. Using Monte Carlo simulations, Teverovsky et al. (1999) show that the larger is the number of lags, the lower is the power of the test for long-range dependence. Andrews (1991) suggests a procedure for selecting the number of autocovariances to include in the calculation of the adjusted RRA, based on the first-order autocorrelation coefficient. Jacobsen (1996) applies the RRA to the residuals of AR(1) and MA(1) models. Fillol and Tripier (2004) use Generalized Least Squares to estimate jointly a short-range dependence parameter, and a generalized Hurst exponent based on the scaling function estimator developed by Mandelbrot et al. (1997). None of these studies, however, provides a clear-cut solution to the problem of discriminating between short-range dependence and long-range dependence.

In the present study, the problem of disentangling short-range dependence and long-range dependence is approached by comparing