Physica A 387 (2008) 6319–6329

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

An ever-closer union? Examining the evolution of linkages of European equity markets via minimum spanning trees

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ARTICLE INFO

Article history:
Received 20 February 2008
Received in revised form 4 June 2008
Available online 16 July 2008

Keywords:
Econophysics
Minimum spanning trees

ABSTRACT

The concept of a minimum spanning tree (MST) is used to study the process of comovements for 21 European Union stock market indices. We show how the minimum spanning tree and its related hierarchical tree evolve over time and describe the dynamics. Over the period studied, 1999–2006, the French equity market provides the main linkages in the system. The 2004 Accession states are more loosely connected to the other markets; they form two groupings, with the Czech Republic, Hungary, and Poland having tighter links to the main markets than the remaining Accession markets. Shorter distances between markets indicate a potential reduction of the benefits of international portfolio diversification in European markets, with the possible exception of those markets at the outer limits of the MST.

1. Introduction

The evolution of the European Union (EU) over the last decade has been marked by such major events as the establishment of the European Monetary Union (EMU), with the introduction of the euro as the single currency for twelve of the EU member states, and an expansion of EU membership in 2004 to include ten new countries, primarily Central and Eastern European (CEE) states. An additional two countries, Romania and Bulgaria, joined the EU at the start of 2007, while Slovenia (2007) and Malta (2008) have adopted the euro. This process of closer economic and financial cooperation has led to increasing levels of financial market integration in the area, as documented in an extensive literature, surveyed in several review studies [1,2]. The effects of these developments on comovements in financial markets are of great importance not only to policymakers but also to investors, because of their potential to affect international asset allocation decisions and diversification benefits. Specifically, the attractiveness of international portfolio diversification will weaken as returns are equalized across countries [3,4].

The present research examines possible effects on the benefits of international portfolio diversification across EU equity markets of both "old" members and "new" Accession countries during the 1999–2006 period. We use a method introduced into the physics literature by Mantegna [5], known as Minimum Spanning Tree (MST) analysis, to examine the extent and evolution of comovements between these EU equity markets. Based on graphing theory, MST analysis provides a parsimonious representation of the network of correlations between markets and is particularly suitable for extracting the most important information concerning linkages when a large number of markets is under examination. A dynamic application allows us to identify the evolution of the patterns of the most important connections between EU equity markets and to examine a number of questions concerning their interrelationships. Which are the "core" EU equity markets? Have

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doi:10.1016/j.physa.2008.07.012

equity market comovements increased substantially due to closer harmonization of economic and fiscal policies associated with the creation of the EMU? What patterns of linkages can be observed for the Accession members' equity markets? Our results indicate increased comovements, identify France rather than Germany as the core market, and describe distinct patterns of linkages of the new members' equity markets.

The paper is organized as follows. Section 2 provides a literature review of the MST methodology, which is discussed in Section 3. The data are described in Section 4. Results are presented in Section 5 and the conclusions in Section 6.

2. Literature review

Minimum spanning tree analysis has been applied previously to study the clustering behavior of individual stocks within a single country, usually the US [6–9]. These studies typically find a strong correspondence between business sector and tree structure, illustrating the ability of the MST methodology to convey meaningful economic information. While these are static approaches, a variety of dynamic MST analyses of the time-varying behavior of stocks has also been developed and applied in Refs. [10–15]. MST analysis has also been applied to the foreign exchange markets as a means to trace the dynamics of relationships between currencies [16].

To date only two studies have been published applying the MST approach to groups of national equity markets. A simple dynamic analysis based on partially overlapping windows of indices for 20 countries for the years 1988–1996 finds that markets group according to a geographical principal, as is also the case for a static examination of 51 world indices for the years 1996–1999 in the same study [17]. Coelho et al. [18] apply dynamic MST methods to examine the time-varying behavior of a group of 53 developed and emerging markets over the years 1997–2006. Confirming the earlier evidence of a geographical organizing principle, this research also finds increasing MST density over time, reflecting higher levels of comovement of international equity markets.

3. Methodology

As proposed by Mantegna [5], linkages between stock returns can be examined by applying a simple transformation of the elements of the correlation matrix of returns into distances. A connected graph is constructed in which the "nodes" correspond to companies (or, as here, stock indices) and the "distances", or "edges", between them are obtained from the appropriate transformation of the correlation coefficients. A minimum spanning tree is generated from the graph by selecting the most important correlations between the stock or index returns. The MST reduces the information space from $N(N-1)/2$ separate correlation coefficients to $(N-1)$ linkages, known as tree "edges", while retaining the salient features of the system.

Specifically, in this study we calculate the correlation matrix of returns of EU member equity market indices and convert the correlations $\rho_{i,j}$ to a distance metric $d_{i,j}$ between each pair of stock indices as follows:

$$d_{i,j} = \sqrt{2(1-\rho_{i,j})}. \tag{1}$$

This forms an $N \times N$ distance matrix D. The distances $d_{i,j}$ vary from 0 to 2, while correlations run from -1 to $+1$. High correlations correspond to small values of $d_{i,j}$. This distance matrix is then used to construct the MST.

The MST is built up by linking all the N elements of the set together in a graph characterized by a minimal distance between nodes (indices). One starts with the pair of elements with the shortest distance (highest correlation). Next, the second-smallest distance is identified and added to the MST. Successive equity markets are added, with the condition that no closed loops are created. The MST is thus a simply connected graph that links all N nodes of the graph with $N-1$ edges such that the sum of all edge weights is a minimum. A static picture of index linkages can be obtained from including the entire data set in a single MST.

In addition to the MST we construct a hierarchical tree to further explore the relationships between the markets. The MST provides the information needed for calculating the subdominant ultrametric distance matrix $D^{<}$, which is used to construct the hierarchical tree. The $D^{<}$ matrix is obtained by defining the distance $d^{<}_{i,j}$ between i and j as the maximum of any Euclidean distance $d_{k,l}$ determined by moving in single steps from i to j through the shortest path connecting i and j in the MST. (For a fuller technical discussion see Mantegna [5].) The hierarchical tree ranks the linkages between markets via the subdominant ultrametric distance, beginning with the pair exhibiting the shortest distance measure. Successive markets are added to the core of this tree in order of increasing distances. Thus, the last markets added to the hierarchical tree are those with the most distant linkages to the "core" market(s).

Several additional tools can be used to further explore the dynamics of the system of EU equity markets. The first two moments of the mean correlations $\rho_{i,j}$ and of the distances $d_{i,j}$ can be presented in rolling-window graphs. The mean correlation coefficient is:

$$\rho = \frac{2}{N(N-1)} \times \sum_{i<j} \rho_{i,j} \tag{2}$$

and its variance is given as

$$\lambda_2 = \frac{2}{N(N-1)} \times \sum_{i<j} (\rho_{i,j} - \rho)^2. \tag{3}$$