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Characterizing bid–ask prices in the Brazilian equity market

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Abstract

This paper presents evidence of long-range dependence in bid–ask prices for individual equity prices in the Brazilian stock market. Moreover, using the Hurst exponent calculated by the Local Whittle method as a measure of long-range dependence, we find evidence supporting that bid–ask prices shows a stronger long-range dependence than the one usually found in closing and opening prices. Finally, we show that bid–ask prices may be characterized by a distribution that decays as a power law reinforcing the results of Plerou et al. [Quantifying fluctuations in market liquidity: analysis of the bid–ask spread, Phys. Rev. E 71 (2005) 046131].

Keywords: Long-range dependence; Bid–ask spreads; Market microstructure; Emerging equity markets; Hurst exponents

1. Introduction

The main function of a given financial market is to provide conditions for the realization of transactions between investors. In this context, the market maker has a fundamental role. Market makers[1] are responsible for setting the so-called bid-price, at which they will buy securities and ask-price, at which they will sell. The difference between these prices is the bid–ask spread, which is the appropriate return for the service provided by the market maker.

One of the main issues in market microstructure research has been to understand the determinants of the bid–ask spread.[2] In particular, Demsetz [3] found that volume, risk and firm size appear to explain most of the variability in the bid–ask spread. On the other hand, Smidt [4] argued that market makers are not simply passive providers of liquidity, but they also respond to fluctuation in their inventory levels. According to Zabel [5], O'Hara and Oldfield [6] and Madhavan and Smidt [7], bid and ask prices are set so as to maximize the present expected value of trading revenue less inventory storage costs over an infinite horizon of trading days.

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[1]Market makers are dealers in the securities exchange who buy and sell securities for their own account to maintain an orderly market in the specific securities they manage.

[2]Detailed review of these issues may be found in Madhavan [1] and O'Hara [2].

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This paper extends two recent papers Cajueiro and Tabak [8,9] that shows that Brazilian equity shares possess long-range dependence and relates these findings to specific financial variables such as leverage, return on equity and market capitalization and to the effect of periodical market closures. In particular, this paper tests bid–ask spreads for long-range dependence in the same dataset used by the previously mentioned papers using the Hurst exponent calculated by the Local Whittle due to Robinson [10] as a measure of long-range dependence. Moreover, it shows evidence that bid–ask prices may be characterized by a distribution that decays as a power law. These two results reinforce the findings of Plerou et al. [11] characterizing the dynamics of bid–ask spreads in the Brazilian stock market.

The rest of the paper is divided as follows. The Local Whittle estimator used here to evaluate the Hurst's exponent is introduced in Section 2. In Sections 3 and 4, a brief overview of the Brazilian equity market and the data are, respectively, presented. In Section 5, the empirical results of this work are exposed. Finally, Section 6 presents some conclusions.

2. The Local Whittle estimator

In this paper the Local Whittle estimator is used to provide the Hurst's exponent. A given variable in a market is said to have long-range dependence with persistent behavior if the Hurst's exponent $H>0.5$, with anti-persistent behavior is if $H<0.5$ and random walk behavior if $H=0.5$.

Let S_t be the bid–ask spread, which is observed at times $t=1,\ldots,n$. Denote by γ_j the lag-j autocovariance of St and by $f(\lambda)$ the spectral density of S_t, such that $\gamma_j = \mathrm{E}((S_0 - E(S_0))(S_j - E(S_j))) = \int_{-\pi}^{\pi}\cos(j\lambda)f(\lambda)\,\mathrm{d}\lambda$.

The Local Whittle estimator is a semi-parametric estimator, which only requires specifying the parametric form of the spectral density when the frequency λ is close to zero,

$$f(\lambda)\sim G(H)|\lambda|^{1-2H} \quad \text{as } \lambda \to 0, \tag{1}$$

when $G(H)$ is a constant. The computation involves an additional parameter m, an integer less than $N/2$, where N is the size of the time series, and such that, as $N\to\infty$,

$$\frac{1}{m}+\frac{m}{N}\to 0. \tag{2}$$

This means that as N gets large, m gets large as well, although slower. For a spectral density of form (1), the Whittle approximation of the Gaussian likelihood function is obtained by minimizing

$$Q(G,H)=\frac{1}{m}\sum_{j=1}^{m}\left(\frac{I(\lambda_j)}{G\lambda_j^{1-2H}}+\log(G\lambda_j^{1-2H})\right), \tag{3}$$

where $\lambda_j = 2\pi j/N$ and $I(\lambda_j)$ is periodogram. So this estimator sums the frequencies only up to $2\pi m/N$.

Replacing above G by its estimate $\hat{G}$,

$$\hat{G}=\frac{1}{m}\sum_{j=1}^{m}\frac{I(\lambda_j)}{\lambda_j^{1-2H}}. \tag{4}$$

One may define

$$R(H)=Q(\hat{G},H)-1=\log\left(\frac{1}{m}\sum_{j=1}^{m}\frac{I(\lambda)}{\lambda_j^{1-2H}}-\frac{2H-1}{m}\sum_{j=1}^{m}\log(\lambda_j)\right). \tag{5}$$

Robinson [10] showed that under certain technical assumptions,

$$\hat{H}=\arg\min R(H), \tag{6}$$

converges in probability to actual value H, i.e.,

$$m^{1/2}(\hat{H}-H)\to_{\mathrm{d}}\mathrm{Normal}(0,1/4). \tag{7}$$