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An economic growth model with endogenous carrying capacity and demographic transition☆

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ABSTRACT

In this paper, a mathematical model is set up to inquire population change under interaction between the economic growth and human population carrying capacity. By introducing the population growth equation with variable carrying capacity into the classical Solow model and combining the population growth equation, we obtain a two-dimensional dynamical system. It is proved that the dynamical system has a unique equilibrium and the solution of the dynamical system is asymptotically stable. By qualitative analysis, we obtain that the population growth rate increases from zero to a positive level firstly and then decreases to zero and per capita capital increases strictly along a normal economic growth path. Therefore, the model implies that the demographic transition appears under the interaction between economic growth and human population carrying capacity.

1. Introduction

The affection of the demographic transition and the carrying capacity (CC) of human population on economic growth has aroused much attention by demographers, economists and biologists, recently [1–11].

In general, the population carrying capacity is assumed to be fixed or change with the time and the growth of population satisfies logistic equation or the equation of logistically variable carrying capacity [12–14]. However, the population carrying capacity expands with the economic growth and the population, and the labor increases, which accelerates the economic growth, as the population carrying capacity expands. So, there exists interaction between the economic growth and the population carrying capacity.

In this paper, we assume that the population carrying capacity increases with the economic growth, and the population grows as the population carrying capacity expands. By integrating the variable population carrying capacity function into the classical Solow model and combining the population growing equation, we obtain a two-dimensional dynamical system. It is proved that the dynamical system has a unique nonzero equilibrium and its solution is asymptotically stable and converges to the equilibrium.

By qualitative analysis, we obtain that the population growth rate increases from zero to a positive level firstly and then decreases to zero, and per capita capital increases strictly along the economic growth path that starting from a point on the curve of the variable population carrying capacity function. Therefore, the model implies that the demographic transition appears under the interaction between economic growth and human population carrying capacity. In the end of this paper, we provide a numerical simulation to show the process of the economic growth and the demographic transition.

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The remainder of the paper is organized as follows. In Section 2, the model is presented. The existence and uniqueness of nonzero equilibrium of the model is proved and the type of the equilibrium is discussed in Section 3 and in Section 4. In Sections 5–7, we analyze the dynamics of the model, per capita capital and the population along the economic growth path. The numerical simulation and some conclusions are presented in Sections 8 and 9.

2. Setup the model

Consider a closed economy with a production sector. The production function, $F(K, L)$ is neoclassical and satisfies Inada conditions. At time t, the net capital investment is

$$\dot{K} = sF(K, L) - \delta K, \tag{1}$$

where s and δ are the saving rate and capital depreciation rate, respectively.

Suppose that $N(t)$ is the number of population of the economy at time t and the carrying capacity increases with the economic growth, i.e., increases with the aggregate capital. Furthermore, we assume that the $N(t)$ satisfies a modified logistic population growth equation [14]

$$\dot{N} = \gamma N \left[1 - \frac{N}{\phi_1(K)}\right],$$

where $\phi_1(K)$ is the variable carrying capacity which grows with the aggregate capital and satisfies $\phi_1(0) > 0$, $\phi_1'(K) > 0$, $\phi_1''(K) < 0$ and $\lim_{K\to\infty} \phi_1'(K) = 0$.

We further assume that the labor participation rate is constant, i.e., $L(t) = \lambda N(t)$, where $\lambda > 0$ is the constant labor participation rate, then we have

$$\dot{L} = \gamma L \left[1 - \frac{L}{\phi(K)}\right], \tag{2}$$

where $\phi(K) = \lambda \phi_1(K)$.

From (1) and (2), we obtain the dynamical system below

$$\dot{K} = sF(K, L) - \delta K, \tag{3}$$

$$\dot{L} = \gamma L \left[1 - \frac{L}{\phi(K)}\right]. \tag{4}$$

3. The existence and uniqueness of the nonzero equilibrium

A point $(K^*, L^*) \in \mathbb{R}_+^2$ is an equilibrium of the dynamical system (3)–(4) if and only if it satisfies the following equations

$$sF(K, L) - \delta K = 0, \tag{5}$$

$$\gamma L \left[1 - \frac{L}{\phi(K)}\right] = 0. \tag{6}$$

Lemma 1. *Eq. (5) decides a line $L = \frac{1}{k^*}K$, on which $\dot{K} = 0$, where k^* is the nonzero solution of the equation $sf(k) - \delta k = 0$ and Eq. (6) decides a curve $L = \phi(K)$ on which $\dot{L} = 0$. The line $L = \frac{1}{k^*}K$ and the curve $L = g(K)$ have a unique intersection (K^*, L^*) which satisfies $k^*\phi'(K^*) < 1$.*

Proof. From $sF(K, L) - \delta K = L[sf(k) - \delta k] = 0$ and $sf(k) - \delta k = 0$, we have a unique nonzero solution k^*, and also $\dot{K} = 0$ on the line $L = \frac{1}{k^*}K$, where $k = \frac{K}{L}$, $f(k) = F\left(\frac{K}{L}, 1\right)$.

Let $g(K) = \frac{1}{k^*}K - \phi(K)$, then from $g(0) = -\phi(0) < 0$,

$$\lim_{K\to+\infty} g(K) = K\left[\frac{1}{k^*} - \frac{\phi(K)}{K}\right] = +\infty,$$

the line $L = \frac{1}{k^*}K$ and the curve $L = g(K)$ have at least an intersection. Since $L = \phi(K)$ is a concave function, the line $L = \frac{1}{k^*}K$ crosses it only once from lower to upper and $\phi'(K) < \frac{1}{k^*}$. This completes the proof of the lemma. □

From Lemma 1, we have

Theorem 1. *The dynamical system (3)–(4) has a unique nonzero equilibrium.*