Insurance: Mathematics and Economics 46 (2010) 317–327

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

A benchmarking approach to optimal asset allocation for insurers and pension funds

Andrew E.B. Lim [a], Bernard Wong [b,*]

[a] Department of Industrial Engineering and Operations Research, University of California, Berkeley, United States

[b] School of Actuarial Studies, University of New South Wales, Australia

ARTICLE INFO

Article history:
Received July 2009
Received in revised form
November 2009
Accepted 3 November 2009

JEL classification:
C61
G11

MSC:
IE13
IE43
IE53
IB10
IB81

Keywords:
Asset–liability management
Portfolio optimization
Benchmarking

ABSTRACT

We solve the optimal asset allocation problem for an insurer or pension fund by using a benchmarking approach. Under this approach the objective is an increasing function of the relative performance of the asset portfolio compared to a benchmark. The benchmark can be, for example, a function of an insurer's liability payments, or the (either contractual or target) payments of a pension fund. The benchmarking approach tolerates but progressively penalizes shortfalls, while at the same time progressively rewards outperformance. Working in a general, possibly non-Markovian setting, a solution to the optimization problem is presented, providing insights into the impact of benchmarking on the resulting optimal portfolio. We further illustrate the results with a detailed example involving an option based benchmark of particular interest to insurers and pension funds, and present closed form solutions.

1. Introduction

Traditional optimal asset allocation problems in the investment management literature typically involves maximizing the expected utility of a terminal portfolio value, and where the utility is a concave function possibly satisfying additional properties. However it is well known (see, for example, Panjer et al. (1998) and Yang and Zhang (2005)) that it is also important to consider the effects of the liability payments when determining the optimal asset allocation for insurers and pension funds. Moreover, adaptation of standard financial economic methods (for example, maximization of expected power utility of terminal surplus) in an insurance/pensions setting is often not straightforward due to the nature of insurance and pension problems. In particular, typical utility functions favored in the financial economics literature (e.g. power utility) often involve a strict floor (typically at zero surplus) in the terminal wealth which cannot be violated (see also Teplá (2001) for a related approach where the solvency guarantee is placed in the constraints). While such absolute solvency guarantees may be a desirable property in some applications, it is well recognized in the insurance literature that such guarantees may not be financially desirable, or, indeed, feasible. For example, in underfunded defined benefit pension plans the assets are not sufficient to cover the liabilities with certainty by definition (see, for example, the discussion in Detemple and Rindisbacher (2008)), and in many insurance models the probabilistic structures of many claim processes often mean that liabilities cannot be funded with certainty under a realistic initial asset wealth, even if a guarantee is achievable at all. Such situations lead to an ill-posed optimization problem if the floor cannot be met with certainty, and no optimal solutions are available.

Hence in insurance and pension optimal asset allocation problems it is vital to consider approaches that can tolerate shortfalls. In this paper we consider the asset allocation problem by using a benchmarking approach. Under this approach the objective is an

* Corresponding address: School of Actuarial Studies, Australian School of Business, University of New South Wales, NSW 2052, Australia. Tel.: +61 2 9385 2827; fax: +61 2 9385 1883.

E-mail addresses: lim@ieor.berkeley.edu (A.E.B. Lim), bernard.wong@unsw.edu.au (B. Wong).

0167-6687/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.insmatheco.2009.11.005

increasing and concave function of the relative performance of the asset portfolio versus a benchmark, possibly with an additional floor if required. In insurance and pension applications the benchmark will be a function of either target, or contractual liability payments. This approach tolerates but progressively penalizes underperformance, while at the same time progressively rewards outperformance. Uncertainty in the financial market will be driven by underlying Brownian motions, while the assets and the benchmark are assumed to be general stochastic processes adapted to the filtration of the Brownian motions. In particular, it is not required that the assets or the benchmark follow geometric Brownian motion (a common assumption in the literature), nor do they in fact need to be Markov, and can depend on other underlying processes in a similar manner to traditional actuarial investment return models. It could also be from a large class of probability distributions. The benchmarks that we consider are very general random variables measurable with respect to the underlying filtration, are not required to be attainable with the investor's current wealth. We present a general solution to the asset allocation problem, and illustrate our results with closed form solutions for an option based benchmark of particular relevance to insurers and pension funds.

Several methods have been proposed in the insurance literature to consider the optimal asset allocation problem with tolerance for possible shortfalls. We note, however, that these methods differ from ours in either the objective function that is used and/or the stochastic model for the assets and liabilities. The objective functions that have been studied in the literature can be broadly classified into two alternative groups. The first group considers as an objective the probability of reaching some desired outcome (for example, solvency). Examples include Browne (1995, 1999, 2000), Josa-Fombellida and Rincón-Zapatero (2006). From a behavioral perspective however this is often undesirable as it implies that the decision maker will solely focus on achieving the desired outcome (for example, to stay solvent), and in particular the level of final outperformance or underperformance is ignored. The second approach is to maximize the expected utility of the surplus. In order to tolerate shortfalls such an approach requires a utility function that can be defined over negative values. The two prime examples of such utility functions are those of exponential and quadratic type. The application of an exponential utility function in dynamic asset–liability modelling was explored recently in Korn and Wiese (2008), Yang and Zhang (2005), Wang (2007) and Wang et al. (2007). Quadratic utility/loss functions have been explored in the insurance literature by Cairns (2000), Chiu and Li (2006), Xie et al. (2008), Chen et al. (2008), and is also related to the problem of mean-variance hedging (see Schweizer (2001), Lim and Zhou (2001), Lim (2004, 2005) and the references therein). While the use of the exponential and quadratic approaches provide a number of benefits including tractability, both have undesirable properties which are well known. Exponential utility, for example, implies constant absolute risk aversion, while quadratic utility possesses a saturation level beyond which utility decreases with increasing wealth. (See Luenberger (1998) for additional discussion).

The use of a benchmarked return in a "utility" function has also been used in the literature for problems related to real returns and also with target pension funding ratios. In particular, Brennan and Xia (2002) and De Jong (2008) considered a dynamic asset allocation problem for long term investors where the objective function is a power transformation of the real wealth, while Cairns et al. (2006) investigate stochastic lifestyling strategies for pension plans by considering as an objective function a power transformation of the terminal wealth divided by terminal salary. Davis and Lleo (2008) considered a risk sensitive asset allocation problem where the objective is related to the power "utility" of the benchmarked return, and where the benchmark is a variant of Geometric Brownian Motion. We note however that the previous papers assume that prices processes and benchmarks are variants of Geometric Brownian motion, and "utility" functions of power type. In contrast, we consider very general dynamics for asset price dynamics and the benchmark, and general increasing concave functions of the relative performance. In particular, benchmarks such as the maximum of a random quantity (such as a stock index) and a minimum return, which are not naturally formulated as Geometric Brownian Motion but are of interest in asset and liability management, can be handled in our model.

In related areas, although not of direct relevance to the problem we consider, "utility" on benchmarked performance have also been used in van Binsbergen et al. (2008) for the modelling of decentralized investment management, and in Lim et al. (forthcoming) for robust asset allocation problems.

Finally, note that the benchmarking approach is also applicable to general investment management problems without a liability. Indeed, it is also particularly suited to problems where the performance of an investment fund is measured in a relative sense, with the benchmark being, for example, the performance of a market index, or a quantile of peer performance, or any combinations thereof.

An outline of our paper follows. In Section 2 we setup the financial market we consider. Section 3 defines a benchmark, and provides discussion regarding the concept of a benchmarking function and its relationship to standard utility functions. A general solution to the optimization problem using martingale techniques is presented in Section 4. A detailed example involving an option based benchmark of particular interest to insurers and pension funds is studied in Section 5. Section 6 concludes.

2. Financial market

We use a standard financial market model setup (cf. Karatzas and Shreve (1998)). Consider the probability space $(\Omega, \mathcal{F}, \mathbf{P})$ and the time interval $[0, T]$, the filtration being generated by k-dimensional Brownian Motion $W_i(\cdot),\ i = 1, 2, \ldots, k$, augmented to satisfy the usual conditions.

2.1. Primary securities

Assume that there are $k+1$ primary securities traded in the time interval $[0, T]$. One of the securities represents a savings account process $S_0(\cdot)$, with dynamics

$$S_0(t) = \exp\left\{\int_0^t r(u)\mathrm{d}u\right\}, \tag{1}$$

where $r(\cdot)$ is a progressively measurable process representing the short rate, with

$$\int_0^T |r(t)|\,\mathrm{d}t < \infty. \tag{2}$$

There are also k other assets (which we will call "stocks" for notational convenience), with a time 0 value of $\{S_i(0) > 0, i = 1, \ldots, k\}$, and dynamics

$$\mathrm{d}S_i(t) = \mu_i(t)S_i(t)\mathrm{d}t + S_i(t)\sum_{j=1}^{k}\sigma_{ij}(t)\mathrm{d}W_j(t) \quad i = 1, 2, \ldots, k, \tag{3}$$

for some progressively measurable, k-dimensional vector valued $\mu(\cdot)$, and $k \times k$-dimensional matrix valued $\sigma(\cdot)$ processes, satisfying

$$\int_0^T \sum_{i=1}^{k} |\mu_i(t)|\,\mathrm{d}t < \infty, \tag{4}$$

$$\int_0^T \sum_{i=1}^{k}\sum_{j=1}^{k}\sigma_{ij}^2(t)\mathrm{d}t < \infty,$$