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A multiscale entropy approach for market efficiency

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ABSTRACT

Motivated by the recently evolutionary economic theories, we propose to study market efficiency from an informational entropy viewpoint. The basic idea is that, rather than being an all-or-none concept as in classic economic theories, market efficiency changes over time and over time horizons. Within this framework, market efficiency is measured in terms of the patterns contained in the price changes sequence relative to the patterns in a random sequence. In line with evolutionary finance ideas, the empirical results for the Dow Jones Index showed that the degree of market efficiency varies over time and is dependent of the time scale. In general, the DJI is more efficient for shorter (about days) than for longer (about months and quarters) time scales. On the other hand, the market efficiency exhibits a cyclic behavior with two dominant periods of about 4.5 and 22 years. It is apparent that the 4.5-year cycle is related to inventory (Kitchin-type) effects, while the 22-year cycle to structure inversion (Kondriatev-type) cycles.

1. Introduction

In his seminal paper, Fama (1970) introduced the term "market efficiency" to refer to the role of information in the formation of prices. The efficient market hypothesis (EMH) establishes that new information is quickly and correctly reflected in its current security price. Within the EMH, arbitrage conditions are quickly eliminated by the action of informed market participants. That is, classical economic logic indicates that as money is brought to bear against a given trading opportunity, any predictable excess returns must be reduced and eventually eliminated (Stein, 2009). An important consequence of the EMH is that price changes must be unforecastable if they fully incorporate the information and expectations of the huge diversity of market participants.

In an informationally efficient market, price dynamics must follow a random walk behavior resulting from the action of market participants attempting to profit from their information. The information is quickly incorporated into market prices, eliminating the profit opportunities that first motivated the trades. Therefore, no profits can be obtained from information-based trading because information on price patterns is evenly distributed; leaving only noise information associated to random price fluctuations. Within these ideas, a huge body of scientific literature on the EMH has focused on showing that prices follow a random walk behavior by studying the predictability of security returns on the basis of past price changes. The reader is referred to the recent survey by Lim and Brooks (2011) for a detailed discussion of the main contributions in the empirical analysis of the random walk hypothesis.

1.1. The adaptive market hypothesis

Recent surveys have showed the existence of a strong discrepancy between proponents and the EMH and advocates of the behavioral finance (Lim & Brooks, 2011; Malkiel, Mullainathan, & Stangle, 2005). It is apparent that this discrepancy arises because the market efficiency is posed as an all-or-none condition. Lo (2004) has proposed that both intellectual positions can be reconciled within an evolutionary framework where, rather than an all-or-none condition, market efficiency is a characteristic that varies continuously over time and across markets. In this regard, Lo (2004) proposed the new paradigm of adaptive market hypothesis (AMH) in which the EMH can coexist alongside behavioral ideas in an intellectually consistent manner (Lim & Brooks, 2011). The underlying idea behind the AMH is that markets behave as an evolutionary system in which participants and instruments interact and evolve dynamically according to intrinsic rules of economic selection. As a consequence, financial agents compete and evolve to survive. In contrast to the classic view, these agents endowed with bounded rationality (Simon, 1955), conform decisions within a highly uncertain environment that deviates the decisions from an optimal fashion (Farmer & Lo, 1999; Giglio, Matsushita, & Da Silva, 2008; Lo, 2004, 2005). In this view, market efficiency is not an all-or-none question, but a reflection of the complex interaction of the market environments (e.g., socioeconomic conditions) with the behavioral structure of the market participants. As a result, the efficiency of the market varies

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both over time and across markets. Departures from "full efficiency" are allowed as the market dynamics adapts to, e.g., changing conditions and external shocks. It has been argued that, under the AMH, convergence to equilibrium (which is central to the EMH) is neither guaranteed nor likely to occur at any time (Lo, 2005).

1.2. The entropy analysis approach

The existence or not of exploitable patterns in price changes has been the keynote for testing market efficiency. The combination of patterns analysis and the capacity of specific agents for processing information should provide a framework for making testable the AMH. From a macroscopic viewpoint, entropy can be related to the number and diversity of patterns and variations that a system can display for a very large set of trajectories.

Although entropy is a powerful concept to characterize the diversity of patterns contained in a time series, its application to analyze financial time series has been constrained to a limited number of research studies. It is apparent that Gulko (1999) proposed firstly the use of entropy concepts to study financial time series by showing that the maximum-entropy formalism, also called informational efficiency, makes the efficient market hypothesis operational and testable. Darbellay and Wuertz (2000) demonstrated the usefulness of entropy concepts to characterize financial time series by showing that the salient advantage of the entropy approach resides in its ability to account for nonlinear dependences. Pincus and Kalman (2004) suggested that approximate entropy algorithm is suitable for analyzing financial time series as it can be applied to very short sequences and can be used as a marker of market stability. Recently, entropy concepts have been used to quantify market efficiency in foreign exchange and stock markets. In contrast to previous approaches focusing on an all-or-none response to the efficiency question, entropy can provide a relative degree of the efficiency of a given market. Oh, Kim, and Eom (2007) used the global foreign exchange market indices in order to study the efficiency of various foreign exchange markets around the market crisis. It was found that the markets with a larger liquidity (e.g., European and North American) have a higher market efficiency than those with a smaller liquidity. Risso (2008) used Shannon entropy concepts on symbolic dynamics of stock indices to show that the probability of having a crash increases as the entropy decreases.

1.3. Our contribution

In pursuit of a method to test for possible changes in market efficiency, we have resorted in the basic practice of market practitioners; namely, the search of exploitable patterns contained in a sequence of price changes (Theodore, 1996). In this spirit, this work uses informational entropy concepts and methods to study market efficiency. Entropy is a basic concept used to quantify disorder and uncertainty of dynamic systems, which can be related to the number and diversity of patterns and variations that a system can display for a very large set of trajectories (Pincus, 1991).

The approximate entropy algorithm (Pincus, 1991) is used in this work and implemented within a rolling window scheme to estimate the entropy content of a time series of a given length. Market efficiency is defined in relative terms of the entropy of random sequences. Specifically, a market efficiency index is defined in terms of the distance to an entropy benchmark as computed for a Gaussian random sequence. The Dow Jones index is taken as a working case to illustrate the applicability of the entropy approach. The results showed that the market efficiency does not evolve only over time, but also over time horizons. The analysis of the market efficiency in terms of entropy concepts for the Dow Jones showed the existence of two cycles with periods of about 4.5 and 23 years. Interestingly, the latter cycle can be related to the occurrence of large US recessions.

2. Approximate entropy

A direct application of entropy concepts requires the availability of infinite data series with infinite accurate precision and resolution. This is not possible in practice since measurements from real systems are sampled with limited resolution ε and finite sampling rate $1/T_s$. To alleviate this situation, approximate entropy statistics have been introduced (Pincus, 1991) to quantify regularity of time series of finite length. The approximate entropy (AE) computations are conceptually simple and are based on the likelihood that templates in the time series that are similar remain similar on next incremental comparisons. Hence, time series with large AE should have high uncertain fluctuations.

An algorithm for entropy computation for finite data can be described as follows. A time series of length N sampled at time intervals T_s, $\{X_i\}=\{x_1, x_2 \ldots, x_N\}$, where $x_i = x(t_0 + iT_s)$, is considered. It is noted that the length N can be related to a time scale $\tau = NT_s$. Two m-dimensional sequence vectors $u^{(m)}(i)=\{x_i, x_{i+1},\ldots, x_{i+m-1}\}$ and $v^{(m)}(j)=\{x_j, x_{j+1},\ldots, x_{j+m-1}\}$, $i \neq j$, $1 \leq i, j \leq N-m+1$, are selected. These vectors $u^{(m)}(i)$ and $v^{(m)}(j)$ are called similar if their distance $d_{u,v}(i,j)=\max\{|u(i+k)-v(j+k)|: 0 \leq k \leq m-1\}$, is smaller than a specified tolerance ε. For each of the $N-m+1$ vectors $u^{(m)}(i)$, the number of similar vectors $v^{(m)}(j)$ is given by measuring their respective distances. If $n_i^{(m)}$ is the number of vectors $v^{(m)}(j)$ similar to $u^{(m)}(i)$, the relative frequency to find a vector $v^{(m)}(j)$ which is similar to $u^{(m)}(i)$ within a tolerance level ε is given by $C_i(m,\varepsilon,\tau)=\frac{n_i^{(m)}}{N-m}$, where $N-m$ is the number of vectors $v^{(m)}(j) \neq u^{(m)}(i)$ that are potentially similar to $u^{(m)}(i)$. Next, one looks at the relative frequency of the logarithm of $C_i(m, \varepsilon, \tau)$, *i. e.*, $\Phi(m,\varepsilon,\tau)=\frac{1}{N-m+1}\sum_{i=1}^{N-m+1}\ln C_i(m,\varepsilon,\tau)$. For finite N, the approximate entropy is estimated by the statistics $E(m,\varepsilon,\tau)=\frac{1}{T_s}[\Phi(m,\varepsilon,\tau)-\Phi(m+1,\varepsilon,\tau)]$. In this way, lower values of $E(m,\varepsilon,\tau)$ reflect more regular time series, while higher values are associated with less predictable (more complex) time series within the time scale τ. The effects of the tolerance ε, data length N and vector dimension m in the performance of the AE computation (Pincus, 1991). Stable statistics were found for $N>500$. On the other hand, the parameters $m=2$ and $\varepsilon=0.15\sigma$, where σ is the standard deviation of the time series, are commonly used in applications.

2.1. Multiscale approximate entropy

One can expect that entropy is scale-dependent, meaning that a signal is more uncertain for certain time-scales and more irregular for others, so that a complete entropy characterization of time-series should consider the variability over a non-trivial range of scales. A multiscale AE method involves two steps: i) A method allowing looking at representations of the system dynamics at different time scales. For a given the time-series $X=\{x_1, x_2 \ldots, x_N\}$, and a low-pass filter procedure $LP(f)$ with cut-off frequency f, obtain the filtered time series as $Y_f = LP(f) \cdot X$, where $Y_f=\{y_{f,1}, y_{f,2},\ldots, y_{f,N}\}$. The new time-series Y_f retains the complexity of the signal X for frequencies smaller than f, or time-scales higher than $\tau = 1/f$. Different low-pass filtering operations $LP(f)$ are available in commercial packages. In this work, as used in technical analysis by market practitioners to obtain long-term trending of financial signals, low-pass filtering is performed with moving-average filters. In this way, the filtered signal is obtained as

$$y_{f,i} = \frac{1}{n}\sum_{j=1}^{n} x_{i+j-1}$$

The time scale is given by $\tau = \Delta t n$ and $f = 1/\tau$. Here, Δt is the sampling period. ii) The quantification of the degree of irregularity of each