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Active portfolio management with benchmarking: A frontier based on alpha

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ABSTRACT

Active portfolio management often involves the objective of selecting a portfolio with minimum tracking error variance (TEV) for some expected gain in return over a benchmark. However, Roll (1992) shows that such portfolios are generally suboptimal because they do not belong to the mean-variance frontier and are thus overly risky. Our paper proposes an appealing method to lessen this suboptimality that involves the objective of selecting a portfolio from the set of portfolios that have minimum TEV for various levels of *ex-ante* alpha, which we refer to as the alpha-TEV frontier. Since practitioners commonly use *ex-post* alpha to assess the performance of managers, the use of this frontier aligns the objectives of managers with how their performance is evaluated. Furthermore, sensible choices of ex-ante alpha lead to the selection of portfolios that are less risky (in variance terms) than the portfolios that active managers would otherwise select.

1. Introduction

Investors often delegate the management of part of their wealth to active portfolio managers (hereafter, 'managers') whose objective is to select a portfolio that is expected to beat the return on a benchmark while having minimum tracking error variance (TEV).[1] However, in an influential paper, Roll (1992) shows that such portfolios do not belong to the *mean-variance frontier*, which consists of portfolios with minimum variance for various levels of expected return. Hence, portfolios selected by managers are generally overly risky for investors.[2]

Previous work advocates three methods to mitigate this tendency of managers to select overly risky portfolios. All of them impose a limit on the amount of risk that managers can take, but differ on the measure of risk used. Specifically, Roll (1992), Jorion (2003), and Alexander and Baptista (2008) use, respectively, beta, variance, and value-at-risk to set these risk limits.[3] Note that these limits are imposed assuming that managers still have mean-TEV objective functions.

Our paper proposes a fourth method that utilizes an alternative objective function without explicitly imposing a risk limit. Specifically, this method involves the objective of selecting a portfolio with some given level of *ex-ante* alpha while still minimizing TEV.[4] Such a method has two desirable features. First, since practitioners (e.g., Morningstar) commonly use Jensen's (1968) *ex-post* alpha to assess the risk-adjusted performance of managers, it aligns the objectives of managers with how their performance is evaluated. Second, sensible choices of ex-ante alpha (hereafter, 'alpha') lead to the selection of portfolios that are less risky (in variance terms) than the portfolios that managers would otherwise select. Since portfolios selected by managers with mean-TEV objective functions are generally overly risky from the perspective of investors, the proposed method benefits investors.

We begin by considering the case when short sales are allowed. Four main contributions to the literature are made. First, we

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[1] A portfolio's *tracking error* is the difference between the portfolio's return and the benchmark's return.

[2] This result relies on the assumption that investors face only portfolio risk. When investors also face background risk (from sources such as labor income), Baptista (2008) shows that there exist conditions under which they can optimally delegate the management of their wealth to managers.

[3] Alexander and Baptista (2006) investigate the impact of adding a drawdown constraint to the mean-TEV model. For an examination of the portfolio selection implications of a shortfall constraint on a manager's tracking error within an expected utility model, see Basak et al. (2006).

[4] A portfolio's *ex-ante* alpha is the difference between its expected excess return and the benchmark's beta-adjusted expected excess return, where excess returns are equal to raw returns minus the risk-free return.

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characterize the *alpha-TEV frontier*, which consists of portfolios that minimize TEV for various levels of alpha. In particular, we show that portfolios on the alpha-TEV frontier exhibit *three-fund separation*, with the funds being two portfolios on the mean-variance frontier and the benchmark. This result is similar to that derived by Roll (1992) for the *mean-TEV frontier*, which consists of portfolios that minimize TEV for various levels of expected return. However, the weights of the three funds in portfolios on the alpha-TEV frontier generally differ from those in portfolios on the mean-TEV frontier. For example, the weight of the benchmark in the former portfolios generally differs from 100%, while in the latter it is 100%.[5]

Second, we show that the alpha-TEV frontier is related to the *beta-constrained mean-TEV frontier* of Roll (1992), which consists of portfolios that, given a beta constraint, minimize TEV for some level of expected return. Specifically, any portfolio on the alpha-TEV frontier is also on the beta-constrained mean-TEV frontier for some beta constraint and level of expected return that depend on its alpha. However, portfolios on the beta-constrained mean-TEV frontier are generally not on the alpha-TEV frontier. Hence, the set of portfolios on one frontier differs from that on the other. For example, while portfolios on the alpha-TEV frontier have different betas, portfolios on the beta-constrained mean-TEV frontier have (by construction) the same beta. Furthermore, as we explain shortly, the alpha-TEV frontier always contains a portfolio that is also on the mean-variance frontier, whereas the beta-constrained mean-TEV frontier generally does not.

Third, we examine whether portfolios on the alpha-TEV frontier are 'closer' to the mean-variance frontier than portfolios on the mean-TEV frontier. A portfolio's *efficiency loss* is defined as its variance minus the variance of the portfolio on the mean-variance frontier with the same expected return. We show that there exists a unique level of alpha for which the correspondent portfolio on the alpha-TEV frontier is also on the mean-variance frontier and thus has a zero efficiency loss. Hence, like portfolios on the mean-TEV frontier, portfolios on the alpha-TEV frontier are generally not on the mean-variance frontier and thus have positive efficiency losses. However, we show that portfolios on the alpha-TEV frontier are closer to the mean-variance frontier than portfolios on the mean-TEV frontier if alpha is strictly between zero and twice the aforementioned unique level of alpha. Thus, when alpha is appropriately chosen, portfolios on the alpha-TEV frontier have smaller efficiency losses than portfolios on the mean-TEV frontier.[6]

Fourth, we provide guidance on how to set alpha when using the alpha-TEV model to achieve any given reduction in efficiency loss (relative to the mean-TEV model). As noted earlier, there exists a unique portfolio on the alpha-TEV frontier that is also on the mean-variance frontier. Hence, there is a unique value of alpha that reduces the efficiency loss by 100%. However, we show that if the desired reduction is less than 100%, then there exist two levels of alpha that lead to such a reduction.

Next, we consider the case when short sales are disallowed (motivation for this case can be found in, e.g., Jagannathan and Ma (2003) and Almazan et al. (2004)). Since no analytical results are available, an example is used to compare the cases when short sales are allowed and disallowed. As when they are allowed, we find that if alpha is appropriately chosen, then portfolios on the alpha-TEV frontier have smaller efficiency losses than portfolios on the mean-TEV frontier. However, the maximum efficiency loss reduction when short sales are disallowed is possibly smaller than 100% because portfolios on the mean-variance frontier when they are allowed may involve short positions.[7] Nevertheless, we find that the efficiency loss reduction arising from using the alpha-TEV model instead of the mean-TEV model can still be notable.

Our paper is related to the literature recognizing that managers may have incentives to take actions that are suboptimal for investors. First, these incentives can be induced explicitly by compensation contracts that are based on the managers' performance relative to a benchmark (see, e.g., Kritzman, 1987; Starks, 1987; Grinblatt and Titman, 1989; Admati and Pfleiderer, 1997; Elton et al., 2003; Goetzmann et al., 2007). Second, the aforementioned incentives can be induced implicitly by the relationship between fund inflows and performance (see, e.g., Gruber, 1996; Chevalier and Ellison, 1997; Sirri and Tufano, 1998; Basak et al., 2007; Sensoy, 2009).[8] Basak et al. (2008) show that adding a tracking error restriction to a manager's expected utility maximization problem can offset these implicit incentives and thus benefit investors.

It can also be suboptimal for a decision-maker to use an investment approach that involves decentralized management. Sharpe (1981) provides objective functions for managers so that this suboptimality is alleviated. Jorion (2003) shows that diversification among multiple managers may still result in an overly risky portfolio. Elton and Gruber (2004) provide conditions under which it is optimal for a decision-maker to instruct managers to select portfolios that are proportional to the appraisal ratios of the available assets.[9] More recently, van Binsbergen et al. (2008) examine how to optimally select a benchmark to reduce decentralization costs.

In related work, Guasoni et al. (2007) derive the set of payoffs with the maximum appraisal ratio for a given set of available assets. Furthermore, they explore how to achieve the maximal appraisal ratio when a manager can use both the assets in the benchmark and derivatives on them. Our paper differs from theirs in four important respects. First, we characterize the set of portfolios on the alpha-TEV frontier. Second, we show that this set is related to the set of portfolios on the beta-constrained mean-TEV frontier. Third, we compare the location of portfolios on the alpha-TEV frontier in mean-variance space relative to that of portfolios on the mean-TEV and mean-variance frontiers. Fourth, we examine how to set alpha so that the portfolio selected with the alpha-TEV model is closer to the mean-variance frontier than portfolios on the mean-TEV frontier.

We proceed as follows. Section 2 presents the model, characterizes the alpha-TEV frontier when short sales are allowed, and examines the efficiency losses of portfolios on it. Section 3 shows that the alpha-TEV model can be used to achieve any given reduction in efficiency loss (relative to the mean-TEV model) when short sales are allowed. Section 4 provides an example to illustrate our theoretical results when short sales are allowed, and explores the case when they are disallowed. Section 5 concludes. Appendix A contains proofs of our theoretical results. Appendix B relates the alpha-TEV and beta-constrained mean-TEV frontiers.

[5] Equivalently, the asset weight deviations from the benchmark associated with a given portfolio on the alpha-TEV frontier do not generally sum to zero, but those associated with a given portfolio on the mean-TEV frontier do sum to zero.

[6] Put differently, the appropriate use of the alpha-TEV objective function mitigates the suboptimality of portfolios selected according to the mean-TEV objective function.

[7] When short sales are disallowed, we still measure a portfolio's efficiency loss relative to the portfolio on the mean-variance frontier when they are allowed with the same expected return. Footnote 36 justifies this convention.

[8] For an examination of the flow-performance relationship in rational markets, see Berk and Green (2004).

[9] An asset's *appraisal ratio* is defined as the asset's alpha divided by its residual risk. Here, alpha and residual risk are determined relative to a factor model. While our paper also uses alpha, it differs from Elton and Gruber (2004) in that, like Roll (1992) and Jorion (2003), we consider the goal of selecting a portfolio with a relatively small TEV.