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A study on the selection of benchmarking paths in DEA

Sungmook Lim [a], Hyerim Bae [b,*], Loo Hay Lee [c]

[a] Division of Business Administration, Korea University, Jochiwon-Eup, Yeongi-Gun, Chungnam 339-700, Republic of Korea
[b] Department of Industrial Engineering, Pusan National University, San 30 Jangjeondong, Geumjeong-gu, Busan 609-735, Republic of Korea
[c] Department of Industrial and Systems Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore

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ABSTRACT

One of the existing DEA methods' limitations noted in the literature lies in the process of benchmarking of reference targets for inefficient DMUs. Difficulties arising in this process can be summarized to three aspects. First, the reference target might be a hypothetical DMU that does not actually exist (it is difficult and indeed unrealistic to learn from such a DMU). Second, the reference set of an inefficient DMU often has multiple efficient DMUs making it difficult to benchmark multiple best-practice DMUs simultaneously. Third, it is quite impossible for an inefficient DMU to achieve its target's efficiency in a single step, especially when the inefficient DMU is far from the efficient frontier. In order to overcome these difficulties, we propose, in place of the selection of benchmarked DMUs on the efficient frontier, a method of selecting effective benchmarking paths that direct an inefficient DMU to its target on the efficient frontier in an implementable and realistic way. The proposed method was designed based on the idea of the context-dependent DEA proposed by Seiford and Zhu (2003). It starts by clustering DMUs into several layers according to their efficiency scores, and then establishes a benchmarking path across the sequence of layers. Among the DMUs in the next layer, the most preferable one is selected as the next benchmark target, based on three criteria: attractiveness, progress, and infeasibility. We tested the proposed method by applying it to the evaluation of the relative efficiency of operations of 26 container terminals located in Asia.

1. Introduction

DEA (Data Envelopment Analysis) is a non-parametric linear programming based technique for assessing the relative efficiencies of a homogeneous set of Decision Making Units (DMUs) having multiple inputs and outputs. DEA has been widely utilized as a tool for evaluating and improving the performance of manufacturing and service operations in many application areas including schools, hospitals, banks, and public organizations. It is now recognized as a general methodology for solving multi-criteria decision-making problems (Talluri, 2000). Many papers on DEA applications have been published: Grosskopf and Moutray (2001) (education programs), Shafer and Byrd (2000) (IT investments), Donthu, Hershberger, and Osmonbekov (2005) (marketing), Mcmullen and Frazier (1998) (assembly-line balancing), Sueyoshi, Ohnishi, and Kinase (1999) (baseball teams), Luo (2003) (banks), Chang (1998) (hospitals), and others, to name only a few.

There are several reasons DEA has been so successfully applied: it is a non-parametric technique that does not require any assumptions on the production function defining the relationship between inputs and outputs; it can consider multiple inputs and outputs simultaneously; it distinguishes efficient DMUs from inefficient ones, and it provides a proper benchmarking plan for inefficient DMUs.

In DEA, the efficiency score is defined as the ratio of the weighted sum of outputs to the weighted sum of inputs. DMUs having an efficiency score of 1 are considered efficient whereas a score of less than 1 implies that the pertinent DMU is inefficient. Each DMU gives weights, obtained by linear programming, to inputs and outputs in a way that maximizes its efficiency score. DEA, in addition to determining the relative efficiency score of a DMU, identifies a set of efficient units that can be utilized as benchmarks for that DMU's improvement. It should be noted, however, that DEA is primarily a diagnostic tool and does not prescribe any reengineering strategies to make inefficient units efficient (Talluri, 2000).

DEA has been applied in diverse areas, and has seen significant methodological advances as well. Ever since Charnes, Cooper, and Rhodes (1978) introduced the basic DEA model (the CCR model) as a new way to measure the efficiency of DMUs, a number of DEA variants have been developed: the BCC model (Banker, Charnes, & Cooper, 1984), which assumes a variable return to scale, the

* Corresponding author. Tel.: +82 51 510 2733.
E-mail address: hrbae@pusan.ac.kr (H. Bae).

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non-radial additive model (Charnes, Cooper, Golany, Seiford, & Stutz, 1985); the Banker and Morey (1986) model, which involves qualitative inputs and outputs, and the Roll and Golany (1989) model, in which input–output weights are restricted to certain ranges of values.

The practical application of DEA requires that a range of procedural issues be examined and resolved, including those that relate to the homogeneity of units under assessment, the input/output set selected, the weights attributed to them, and the reference targets to benchmark. Among these, the present study focused on the benchmarking of reference targets for inefficient DMUs, the purpose of this paper being to suggest a new method by which inefficient DMUs can reach benchmark targets.

Difficulties arising in benchmarking reference targets for an inefficient DMU can be summarized to three aspects. First, the reference target might be a hypothetical DMU that does not actually exist. The efficiency of an inefficient DMU is measured relative to an efficient DMU or a convex combination of DMUs on the efficient frontier, which serves as a target for improvement. However, the target, when given as a combination of efficient DMUs, is not an actually existing DMU, but a hypothetical one, and it is difficult, indeed unrealistic, to learn from a DMU that does not exist. Second, it is not easy to benchmark multiple best-practice DMUs in the reference set simultaneously. The efficient DMUs that have positive weights in the combination yielding the hypothetical target DMU comprise the reference set of an inefficient DMU. And when the reference set of an inefficient DMU has multiple efficient DMUs, the inefficient DMU has multiple targets to benchmark, which is an obviously confounding situation. Third, it is very likely to be practically infeasible for an inefficient DMU to achieve the target's efficiency in a single step. That is, if the inefficient DMU is far from the efficient frontier, it will be impossible to reach the frontier in a single move, the more reasonable alternative being to make stepwise gradual improvements in getting to the target.

In this paper, we propose an effective benchmarking path selection method that overcomes the DEA limitations discussed above by directing an inefficient DMU to its ultimate target in an implementable and realistic way. The proposed method was devised based on the idea of the context-dependent DEA proposed by Seiford and Zhu (2003). It starts by clustering DMUs into several layers according to their efficiency scores, and then establishes a benchmarking path across the sequence of layers. Among the DMUs in the next layer, the most preferable one is selected as the next benchmark target, based on three criteria: attractiveness, progress, and infeasibility.

This paper is organized as follows. Section 2 provides a brief overview of DEA methodology and the benchmarking difficulties of present concern. Section 3 presents the proposed method and considers the application-prerequisite issues. Section 4 provides details of an empirical study on the proposed method, in which the relative efficiencies of 26 container terminals located in Asia were evaluated. Section 5 summarizes our work and suggests directions for future research.

2. DEA and benchmarking

2.1. Data Envelopment Analysis

DEA evaluates the relative efficiencies of a homogeneous set of DMUs having multiple inputs and outputs. When there are n DMUs utilizing m inputs and producing s outputs, the relative efficiency score of a test DMU k is obtained by solving the following linear programming model proposed by Charnes et al. (1978):

$$
\begin{aligned}
\max_{u,v}\ & \sum_{r=1}^{s} v_r y_{rk} \\
\text{s.t.}\ & \sum_{i=1}^{m} u_i x_{ik} = 1, \\
& \sum_{r=1}^{s} v_r y_{rj} - \sum_{i=1}^{m} u_i x_{ij} \leqslant 0, \quad \forall\, j, \\
& u_i \geqslant \varepsilon, \quad v_r \geqslant \varepsilon, \quad \forall\, i, r,
\end{aligned} \tag{P}
$$

where y_{rj} is the amount of output r yielded by DMU j, x_{ij} is the amount of input i consumed by DMU j, v_r is the weight given to output r, u_i is the weight given to input i, and ε is a positive non-Archimedean infinitesimal. Altering the test DMU sequentially, this problem is solved n times to obtain the relative efficiency scores for all DMUs. Each DMU selects its input and output weights that maximize its efficiency score. Efficiency scores are less than or equal to 1, and a DMU is considered to be efficient if it has a score of 1; otherwise, it is inefficient. For every inefficient DMU, DEA identifies a set of corresponding efficient units (this set is also referred to as the reference set) that can be utilized as benchmarks for improvement. The benchmarks can be obtained from the dual linear program

$$
\begin{aligned}
\min_{\theta,\lambda,s^-,s^+}\ & \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
\text{s.t.}\ & \sum_{j=1}^{n} \lambda_j x_{ij} - \theta x_{ik} + s_i^- = 0, \quad \forall\, i, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - y_{rk} - s_r^+ = 0, \quad \forall\, r, \\
& \lambda_j, s_i^-, s_r^+ \geqslant 0, \quad \forall\, i, j, r,
\end{aligned} \tag{D}
$$

where θ is the efficiency score and λ is the dual variable. By solving (D), we can identify a composite DMU (a linear combination of DMUs) that utilizes less input than the test DMU while maintaining at least the same output levels. The optimal values of the dual variable λ are the coefficients for this linear combination of units. The units involved in the construction of the composite DMU can be utilized as benchmarks for improvement of the inefficient test DMU.

The model described above is known as the CCR model, the most basic DEA model, which assumes a constant return to scale. The BCC model proposed by Banker et al. (1984), a slightly modified version of the CCR model, assumes a variable return to scale. When we add a constraint on the convexity of weights given to inputs and outputs to the CCR model, we can obtain the BCC model. Basically the CCR and BCC models are oriented models that consider efficiency improvement in either the input or the output space, but not in both. Contrastingly, the additive model proposed by Charnes et al. (1985) is a non-oriented DEA model that allows for simultaneous changes of inputs and outputs. Recently, Cooper, Park, and Pastor (1999) proposed a variation of the additive model, referred to as the additive model with range adjusted measure (RAM). Since our proposed method is based on the additive model with RAM, we further discuss the latter here in order to facilitate the explanation of our method in Section 3. The main reason behind our choice of this model is that it allows for the simultaneous consideration of input and output improvement. The additive model with RAM is represented by the linear programming formulation

$$
\begin{aligned}
\max_{s^-,s^+,\lambda}\ & \frac{1}{m+s}\left(\sum_{i=1}^{m} \frac{s_i^-}{R_i^-} + \sum_{r=1}^{s} \frac{s_r^+}{R_r^+}\right) \\
\text{s.t.}\ & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = x_{ik}, \quad \forall i,
\end{aligned}
$$