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Modeling the birth of a liquid market

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Abstract

A continuum market dynamics model with a variable number of traders is proposed. It includes an "impatience" factor that characterizes the frequency of leaving the market by those traders who are not been able to find their counterparts. The market liquidity is defined simply as the presence of traders on both the bid and offer sides of the market. If the price variation is neglected, the deterministic model can be transformed into the Schrodinger equation with a Morse-type potential. It is concluded that the discrete model may be more appropriate for describing a transition to a liquid market. Results of stochastic modeling the birth of a liquid market are discussed.

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1. Introduction

Advances of the Internet technology have been promoting the creation of new electronic markets. The critical problem for an emerging market is the maintenance of its liquidity. The market liquidity can be described in various ways, in particular, in terms of the bid/offer spread and the market depth [1–3]. The minimal criterion of the market liquidity is the very presence of traders on both the bid and offer sides. In this communication, we formulate the model able to describe the process of establishing liquidity in an emerging market. Several models of the market dynamics derived in terms of different trader strategies have been discussed in the literature [4–9] (for a recent review on the agent based computational finance, see Ref. [10]). Usually, these models assume a constant total number of traders partitioned dynamically into different behavior groups, e.g. "chartists" and "fundamentalists". A combinatorial partitioning model with a variable number of traders was recently described in Ref. [11]. In the next

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section, we propose a continuum dynamics model with a variable number of traders in terms of *observable* variables [1]. Its deterministic properties are described in Section 3. It is concluded that the discrete model may be more appropriate for describing a transition to a liquid market. Examples of stochastic modeling of the birth of a liquid market are discussed in Section 4.

2. Model of trader dynamics

We derive a continuum market dynamics model in terms of the numbers of buyers (n_+) and sellers (n_-):

$$\mathrm{d}n_+/\mathrm{d}t = v_{+-}n_- - v_{-+}n_+ + \Sigma R_{+i} + \rho_+ , \tag{1}$$

$$\mathrm{d}n_-/\mathrm{d}t = v_{-+}n_+ - v_{+-}n_- + \Sigma R_{-i} + \rho_- . \tag{2}$$

The functions v_{+-} and v_{-+} describe the probabilities of transfer from sellers to buyers and back, respectively. These functions are determined by the trader strategies [7–9]

$$v_{+-} = v\exp(U), \quad v_{-+} = v\exp(-U), \quad U = \alpha p^{-1}\,\mathrm{d}p/\mathrm{d}t + \beta(p_\mathrm{f} - p) . \tag{3}$$

The first term in the utility function, U, characterizes the "chartist" behavior, i.e., buying while the price of an asset grows and selling when the price falls. The second term describes the "fundamentalist" pattern that promotes buying or selling depending on whether the current price is lower or higher than the fundamental value of the asset. The utility function may also include the terms responsible for the risk-free asset allocation and the peer pressure [7,8]. In Eq. (3), p_f is the fundamental price; α, β, and v are the coefficients that define specifics of the trader behavior.

The functions $R\pm_i\,(i=1,2,\ldots,M)$ and $\rho\pm$ are the deterministic and stochastic rates of changing the *total* number of traders ($n = n_+ + n_-$), respectively. Here we discern three deterministic causes for changing the total number of traders. First, we assume that some traders stop trading immediately after completing a trade as they have limited resources and/or need some time for making new decisions:

$$R_{+1} = R_{-1} = -bn_+n_-, \quad b > 0 . \tag{4}$$

Also, we assume that some current traders and a number of "newcomers" informed by the current traders will be coming in the market. Therefore, the inflow of traders is proportional to the current number of traders

$$R_{+2} = R_{-2} = a(n_+ + n_-), \quad a > 0 . \tag{5}$$

[1] We define the observable variables as those that can be derived from the market transaction records. In particular, they are the instantaneous price, the trading volumes, and the numbers of buyers and sellers. However, the number of the traders with a particular trading behavior (e.g. "fundamentalists") may not be an observable variable (see Discussion in Ref. [12]).