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Probabilistic dominance criteria for comparing uncertain alternatives: A tutorial☆

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Abstract

This paper is a tutorial which demonstrates the current state-of-the-art methods for incorporating risk into project selection decision making. The projects under consideration might be R&D, IT, or other capital expenditure programs. We will show six decision making methods: 1. mean-variance (MV), 2. mean-semivariance, 3. mean-critical probability, 4. stochastic dominance, 5. almost stochastic dominance (ASD), and 6. mean-Gini. We will also describe the assumptions about the risk attitudes of the decision maker which are associated with each of the techniques. While all these methods have been previously applied elsewhere, this is the first paper which shows all of their applications in the project selection context, together with their interrelationships, strengths and weaknesses. We have applied all six techniques to the same group of five hypothetical projects and evaluated the resulting nondominated sets. Among the methods reviewed here, stochastic dominance is recommended because it requires the least restrictive assumptions. ASD and mean-Gini are recommended when stochastic dominance is not practical or when it does not yield definitive choices. MV, mean-semivariance, and mean-critical probability are shown to be flawed.

Keywords: Decision analysis; Risk; Risk analysis; Technology management

1. Introduction

Major project decision making, whether it involves R&D, IT, or other capital spending, is inherently risky. Many R&D projects fail in the research or marketing stage and, typically, only a few succeed. Many IT projects fail in the implementation stage. In making decisions about which projects to include in a company's portfolio, therefore, evaluating risk is one of the most important elements. What are the best, state-of-the-art methods for evaluating project risk?

This paper attempts to answer this question by reviewing a number of methods of characterizing risk in project portfolio decision making, and by showing the comparative strengths and weaknesses of these techniques. While each of these methods has been applied previously in other settings, this is the first paper which shows all of their applications, together with their interrelationships, in the project selection context. We provide a review of probability-based methods which employ dominance criteria in their evaluation. We will cover the methods in the following sequence:

1. *Mean-variance* (*MV*), and 2. *mean-semivariance*: We will show the limitations of these two techniques, for example, that MV assigns the same weight to positive and negative deviations, failing to take skewness into account. Mean-semivariance, which corrects these

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problems of MV, introduces its own problems, for example, requiring the selection of a critical point which can be arbitrary. We will also discuss the assumptions of MV, which place limitations on the decision maker's utility function or on the probability distributions of returns.

3. *Mean-critical probability*: This is an improvement on the above in that it takes probability into account, but it has limitations similar to mean-semivariance, and still requires the selection of a critical value. Further it considers only a single point from the probability distribution.

4. *Stochastic dominance*: This is a major improvement over the first three methods described above in that it takes into account the entire probability distribution of returns for each project. This is, in a sense, a generalization of the mean-critical-probability method which considers only a single parameter and a single point from the distribution. And as noted, MV and mean-semivariance consider only two parameters from the distribution. Stochastic dominance has been used in a wide variety of applications, including, for example, financial portfolio analysis Briec and Kerstens [1] and contract evaluation Ward and Chapman [2].

5. *Almost stochastic dominance* (ASD): This is a relaxation of the requirements of ordinary stochastic dominance. It corrects for certain conditions where ordinary stochastic dominance yields results inconsistent with the common sense preferences of most decision makers.

6. *Mean-Gini*: Finally, mean-Gini analysis provides a method which can be used to compare large numbers of projects. The first five methods which we describe are suitable only for pairwise comparisons of projects. Mean-Gini analysis, as we will see, enables us to compare a large number of projects, while satisfying the necessary (and in some cases sufficient) conditions for second degree stochastic dominance (SSD).

2. Elementary risk measures

To begin our review of risk-evaluation methods we first need to present quantitative statements of the elementary measures of risk. The most basic measure of risk is standard deviation or variance, as shown in Eq. (1) below, where s^2 is the variance, x is a random variable which represents the returns of the project, and $f(x)$ is the probability distribution of returns. One problem with variance is that it gives equal

$$s^2 = \int_{-\infty}^{+\infty} (x - \overline{x})^2 f(x)\,dx, \tag{1}$$

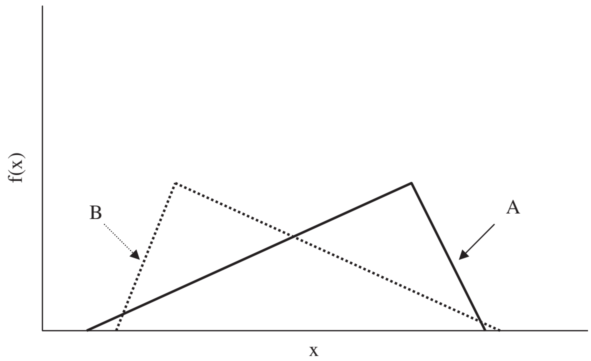

Fig. 1. PDFs of two projects with the same mean and variance where A would be preferred to B by most decision makers (see [4]).

weight to positive and negative deviations from the mean, whereas the decision maker may actually be concerned only about negative deviations. If the probability distribution of the returns for each of the projects is symmetric this creates no problems, but the distribution may not be symmetric. As Bunn [3] points out, the variance measure does not take skewness into account. The weakness of the variance measure is plainly evident in Fig. 1, which shows two distributions of equal mean and variance, where one would clearly be preferred to the other by most decision makers.

To address this concern Bunn also discusses the semivariance as a measure of risk. It is shown in Eq. (2) below, where c is some critical value (chosen by the decision maker) below which we are concerned about deviations. This measure quantifies the variance below that critical value:

$$s_S^2 = \int_{-\infty}^{c} (x - c)^2 f(x)\,dx. \tag{2}$$

Markowitz [4] discusses the limitations of semivariance: "... as far as I know, to date no one has determined whether there is a substantial class of utility functions for which mean semivariance succeeds while MV fails to provide an adequate approximation to EU (expected utility)." Yet another problem for this method is that the choice of a critical value can be arbitrary. Even for a single decision maker, there is no rigorous and unambiguous method for establishing this critical value.

Critical probability (also known as Roy's risk measure [5]) is yet another measure of risk. This measure compares two probability distributions according to the amount of area that falls below some critical value (see Eq. (3) below). Here we are simply concerned about